Phenomenological Formulae for Quarks, Baryons and Mesons

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Abstract

We assume that there is only one unflavored elementary quark family ϵ with three colors and two isospin states (ϵ_u with $I_z=\frac{1}{2}$ and $Q=\frac{2}{3}$, ϵ_d with $I_z=\frac{-1}{2}$ and $Q=\frac{-1}{3}$) in the vacuum. Using phenomenological formulae, we can deduce the rest masses and intrinsic quantum numbers (I, S, C, b and Q) of the excited quarks, from the symmetries of the regular rhombic dodecahedron. The five deduced ground quarks correspond to the five current quarks [u(313)↔u, d(313)↔d, d_S(493)↔s, u_C(1753)↔c and d_b(4913)↔b]. We can then deduce the baryon spectrum and the meson spectrum, from the excited quarks, using sum laws and the phenomenological binding energy formulae of baryons (qqq) and mesons ($q\overline{q}$). The deduced intrinsic quantum numbers (I, S, C, b and Q) of the baryons and the mesons are the same as those of the experimental results. The deduced rest masses of the baryons and the mesons agree with about 98% of the experimental results. Experiments have already discovered almost all of the deduced quarks in Table 11. This paper infers that there are large constant binding energies (>>M_{Proton}) in baryons and mesons. These large binding energies provide a possible foundation for the confinement of the quarks. This paper predicts many new hadrons ($\Lambda_c^+(6599)$, $\Lambda_b^0(9959)$, D(6231), B(9503) and $\Upsilon(17868)$) and a "fine structure" phenomenon of baryons and mesons with large widths. The experimental investigation of the "fine structure" (for example $f_0(600)$ with Γ= 600-1000) provides a crucial test. PACS Numbers: 12.39.-x, 14.65.-q, 14.40.-n, 14.20.-c

I Introduction

The Quark Model [1] has been in use for 40 years. Already it has greatly advanced and pushed forward the particle physics. During the past 40 years, experimental physicists have discovered many new baryons and new mesons with the Quark Model guidance. Today we know of more than three times the number of new particles [2] than we knew of 40 years ago [3]. We think that many new particles will be found, as the development of the experimental equipment and techniques advances, just as astronomers have discovered many new stars after developing more powerful telescopes. In order to account for new experimental discoveries and to advance new experiments, original theoretical models usually need modification. In this paper, we will use phenomenological formulae to modify the current Quark Model. The following problems of the current Quark Model will be addressed in this paper.

A What Problems Need to Be Solved?

A1. A formula that can deduce the rest masses of the quarks is necessary; a formula that can deduce the rest masses of all baryons from the rest masses of the quarks and a formula that can deduce the rest masses of all mesons from the rest masses of the quarks are also necessary. Unfortunately, there is no united mass formula that can deduce the rest masses of the quarks, the baryons or the mesons in the current Quark Model. There is not even a phenomenological formula in the elated literature that can do so.

A2. In order to explain the masses of the light unflavored mesons (η, ω, ϕ, h and f) with I = 0, the Quark Model has to depart from the principle that a meson is made of a quark and an antiquark and allow a meson to be a mixture of three quark-antiquark pairs (u$\overline{u}$, d$\overline{d}$ and s$\overline{s}$). (Note: the meson is not a superposition of three quark-antiquark pairs (u$\overline{u}$, d$\overline{d}$ and s$\overline{s}$) because the three pairs are independent elementary particle pairs

in the current Quark Model). For example [4]:

$$\begin{array}{|l|}\hline \eta(548) = \eta_8 \cos\theta_p \text{-} \eta_1 \sin\theta_p, \\ \hline \eta'(958) = \eta_8 \sin\theta_p + \eta_1 \cos\theta_p, \\ \hline \eta_1 = (\mathrm{u}\overline{u} + \mathrm{d}\overline{d} + \mathrm{s}\overline{s})/\sqrt{3}, \\ \hline \eta_8 = (\mathrm{u}\overline{u} + \mathrm{d}\overline{d} \text{-} 2\mathrm{s}\overline{s})/\sqrt{6}. \\ \hline \end{array} \qquad (1)$$

This "mixture" violates the principle that a meson is made of a quark and an antiquark. It is difficult to understand what physical force could unite the three different independent quark pairs to form one meson. This also causes the model to need a parameter θ_p. There are more than 20 such mixture mesons [ω(782), ϕ(1020)], [h_1(1170), h_1(1380)], ... , [f_2(1810),f_2(2010)] [4]. Therefore, according to the principle that a meson is made of a quark and an antiquark, more quarks are needed to compose these "mixture" mesons. This paper will show that we actually have the quarks to compose these mesons without the three independent-quark-pair mixture.

A3. According to the current Quark Model, there are 12 experimental heavy mesons that are composed of $b\overline{b}$ [5]:

Table 1. The Heavy Mesons of $b\overline{b}$

$q_i\overline{q_j}$	Meson(M)	Γ	$I^G(J^{PC})$	*	$q_i\overline{q_j}$	Meson(M)	$I^G(J^{PC})$
$b\overline{b}$	Υ(1S)(9460)	53kev	$0^-(1^{--})$	*	$b\overline{b}$	χ_{b0}(1P) (9860)	$0^+(0^{++})$
$b\overline{b}$	Υ(2S)(10023))	44kev	$0^-(1^{--})$	*	$b\overline{b}$	χ_{b1}(1P) (9893)	$0^+(1^{++})$
$b\overline{b}$	Υ(3S)(10355)	26kev	$0^-(1^{--})$	*	$b\overline{b}$	χ_{b2}(1P)(9913)	$0^+(2^{++})$
$b\overline{b}$	Υ(4S)(10580)	14Mev	$0^-(1^{--})$	*	$b\overline{b}$	χ_{b0}(2P)(10232)	$0^+(0^{++})$
$b\overline{b}$	Υ(10860)	110Mev	$0^-(1^{--})$	*	$b\overline{b}$	χ_{b1}(2P)(10255)	$0^+(1^{++})$
$b\overline{b}$	Υ(11020)	79Mev	$0^-(1^{--})$	*	$b\overline{b}$	χ_{b2}(2P)(10269)	$0^+(2^{++})$

The mesons Υ(1S)(9460), Υ(2S)(10023), Υ(3S)(10355), Υ(4S)(10580), Υ(10860) and Υ(11020), all have the same $I^G(J^{PC}) = 0^-(1^{--})$ showing that these mesons are not all $b\overline{b}$ with different $I^G(J^{PC})$. They need more quarks to be explained. This paper will show that there are other quarks that can explain these mesons.

A4. The intrinsic quantum numbers (I, S, C, b and Q) of the quarks need to be deduced [6]. This paper will do so using phenomenological formulae [(6), (8), (13) and (14)] from the symmetries of the regular rhombic dodecahedron.

A5. The s-quark, the c-quark and the b-quark can only compose unstable baryons and mesons, giving reason to doubt that these quark are independent elementary particles. Experimental results show that higher mass quarks can decay into lower mass ones, such as b→c , c→s and s→u (or s→d). This might indicate that the five quarks (u, d, s, c and b) are not all independent elementary particles. This paper will show that there may be only two elementary quarks [ϵ_u and ϵ_d] in the vacuum and that all quarks inside hadrons are excited states of these two elementary quarks.

A6. Why do SU(3) and SU(4) work well, and how many quarks will there be inside hadrons? This paper attempts to explain.

A7. A reduction of the number of parameters in the Quark Model is necessary [7].

A8. There are very large full widths in some baryons and mesons, such as Γ = 600-1000 of $f_0(600)$, Γ = 360 of $h_1(1170)$, Γ = 200-600 of $\pi(1300)$ and Γ = 450 of N(2190). The very large full widths need an explanation.

With all of the above problems, a modification of the Quark Model is very difficult to imagine. On one hand, it needs to increase many high mass quarks to explain the high mass baryons and mesons; on the other hand, it needs to reduce the quark numbers to decrease the number of parameters. At the same time, it needs to deduce the intrinsic quantum numbers and the rest masses of the quarks, baryons and mesons. At first, it looks almost impossible.

These high mass baryons and new mesons, however, not only require a further development of the Quark Model, but also create the conditions for this development. On the basis of the experimental baryon and meson spectra, we have found some phenomenological formulae that can reduce the number of elementary quarks and increase the number of excited (from the vacuum) quarks that compose baryons and mesons, as

well as deduce the rest masses and intrinsic quantum numbers (I, S, C, b and Q) of the quarks, baryons and mesons.

B How to Solve the Above Problems

B1. Using only one unflavored (S = C = b = 0) elementary quark family (ϵ) with three colors and two isospin states (ϵ_u wit $I_z=\frac{1}{2}$ and $Q = \frac{2}{3}$, ϵ_d with $I_z=\frac{-1}{2}$ and $Q = \frac{-1}{3}$) in the vacuum and a phenomenological mass formula (37) as well as the symmetries of a regular rhombic dodecahedron (see Fig. 1), we can deduce an excited quark spectrum (m, I, S, C, b and Q), shown in Table 10 and Table 11. The rest masses of the excited quarks are deduced with the mass formula (37). The quantum numbers are deduced with the symmetries of the regular rhombic dodecahedron [(6), (8), (13), (14), (20) and (21)]. The electric charge Q of the excited quarks is determined by the elementary quark [ϵ_u or ϵ_d]. $Q = \frac{2}{3}$ for the excited states of the ϵ_u; $Q = \frac{-1}{3}$ for the excited states of the ϵ_d.

B2. Using the sum laws (48), we can deduced the intrinsic quantum numbers (I, S, C, b and Q) of all known baryons (qqq), from the quark spectrum. The deduced quantum numbers of the baryons match the experimental results. With the sum law (54), we can also deduce the rest masses of the baryons, except for charmed baryons. We can deduce the masses of the charmed baryons using a phenomenological binding energy formula (55). The deduced rest masses of all known baryons (see Table 16 - Table 21) are about 98% consistent with experimental results .

B3. Using the sum laws and the intrinsic quantum numbers (I, S, C, b and Q) of the quarks, we deduce the intrinsic quantum numbers of all discovered mesons ($q_i\overline{q_j}$). The deduced quantum numbers of the mesons are exactly the same as the experimental results. Using a phenomenological binding energy formula (59), we deduce the rest masses of all discovered mesons (see Table 25-Table 31). The deduced masses of the mesons agree within a 2% error of the experimental results.

C Experimental Evidence, Predictions and Crucial Test

This paper deduces many new excited quarks shown in Table 11. Already experiments have discovered almost all of the new quarks inside the baryons or the mesons (see Table 16–Table 21 and Table 25–Table 31). At the same time, this paper predicts many new baryons (Λ_c^+(6599) and Λ_b^0(9959), ...) and new mesons (D(6231), B(9503), Υ(17868), ...). From the large full widths of some mesons and baryons, it also predicts a "fine structure" phenomenon in particle physics. Several mesons or baryons (different constitutions of quarks) that have the same intrinsic quantum numbers (I, S, C, b and Q), the same angular momentums and parities, but different rest masses form a meson (baryon) with large width. The experimental investigations of the "fine structure" (for example f_0(600) with Γ= 600-1000 [2]), provide a crucial test for our phenomenological formulae.

Now let us present the applicable phenomenological formulae.

II Phenomenological Formulae

1. We assume that there is only one unflavored elementary quark family (ϵ) with three colors that have two isospin states (ϵ_u with $I_Z = \frac{1}{2}$ and $Q = \frac{2}{3}$, ϵ_d with $I_Z = \frac{-1}{2}$ and $Q = -\frac{1}{3}$) for each color in the vacuum. Thus there are six Fermi ($s = \frac{1}{2}$) elementary quarks in the vacuum ($S = C = b = 0$).

2. As a colored elementary quark ϵ (ϵ_u or ϵ_d) is excited from the vacuum, its color, electric charge and spin remain unchanged, but it receives energy and intrinsic quantum numbers. In order to explain the experimental results related to quarks-baryons-mesons, we propose a phenomenological formula to determine the strong interaction excited energy of a colored (red, yellow or blue) elementary quark [8], as follows:

$$E(\vec{k},\vec{n}) = V_0 + \alpha[(n_1-\xi)^2+(n_2-\eta)^2+(n_3-\zeta)^2] \qquad (2)$$

where $\vec{k} = (\xi, \eta, \zeta)$ is a vector in a regular rhombic dodecahedron (see Fig. 1) (including its surfaces) in $\vec{k}$ -space. V_0 is the minimum energy that a elementary quark ϵ is excited from the vacuum. α is a constant. n_1, n_2 and n_3 are integers. This formula is the same for any colored elementary quark, so we do not mark the quark's color.

In order to satisfy the symmetries of the regular rhombic dodecahedron, we give a condition for the $\overrightarrow{n} = (\mathrm{n}_1, \mathrm{n}_2, \mathrm{n}_3)$ values. If assuming $\mathrm{n}_1 = l_2 + l_3$, $\mathrm{n}_2 = l_3 + l_1$ and $\mathrm{n}_3 = l_1 + l_2$, we have

$$\begin{aligned} l_1 &= \tfrac{1}{2}(\text{-}\mathrm{n}_1 + \mathrm{n}_2 + \mathrm{n}_3), \\ l_2 &= \tfrac{1}{2}(+\mathrm{n}_1 - \mathrm{n}_2 + \mathrm{n}_3), \\ l_3 &= \tfrac{1}{2}(+\mathrm{n}_1 + \mathrm{n}_2 - \mathrm{n}_3). \end{aligned} \tag{3}$$

The condition is that only those values of $\overrightarrow{n} = (\mathrm{n}_1, \mathrm{n}_2, \mathrm{n}_3)$ are allowed that make $\overrightarrow{l} =$ $(l_1,\ l_2,\ l_3\ )$ an integer vector. For example, $\vec{n}$ cannot take the values $(0, 0, 1)$ or $(1, 1, -1)$, but can take $(0, 0, 2)$ and $(1, -1, 2)$. This is a result of the symmetries of the regular rhombic dodecahedron. The low level allowed $\mathrm{n} = (\mathrm{n}_1, \mathrm{n}_2, \mathrm{n}_3)$ values are shown in (66) of Appendix A.

3. The energy (2) of excited quarks satisfies the symmetries of the regular rhombic dodecahedron. From Fig. 1, we can see that there are four kinds of symmetry points (Γ, H, P and N) and six kinds of symmetry axes (Δ, Λ, Σ, D, F and G) in the regular rhombic dodecahedron. The coordinates (ξ, η, ς) of the symmetry points are as follows:

$$\Gamma = (0,\ 0,\ 0),\ \mathrm{H} = (0,\ 0,\ 1),\ \mathrm{P} = (\frac{1}{2}, \frac{1}{2}, \frac{1}{2}) \text{ and } \mathrm{N} = (\frac{1}{2}, \frac{1}{2}, 0). \tag{4}$$

The coordinates (ξ, η, ς) of the symmetry axes are:

$$\begin{array}{ll} \Delta = (0,\ 0,\ \zeta),\ 0 < \zeta < 1; & \Lambda = (\xi,\ \xi,\ \xi),\ 0 < \xi < \frac{1}{2}; \\ \Sigma = (\xi,\ \xi,\ 0),\ 0 < \xi < \frac{1}{2}; & \mathrm{D} = (\frac{1}{2}, \frac{1}{2}, \xi),\ 0 < \xi < \frac{1}{2}; \\ \mathrm{G} = (\xi,\ 1\text{-}\xi,\ 0),\ \frac{1}{2} < \xi < 1; & \mathrm{F} = (\xi,\ \xi,\ 1\text{-}\xi),\ 0 < \xi < \frac{1}{2}. \end{array} \tag{5}$$

Thus, the energy of the excited quarks has six kinds of symmetry axes (see Table A2). The energy with a $\overrightarrow{n} = (\mathrm{n}_1, \mathrm{n}_2, \mathrm{n}_3)$ along a symmetry axis (from the lowest energy to

the highest energy) forms an energy band. The energy bands on different symmetry axes have different symmetries with different energies and intrinsic quantum numbers (S, C, b and I) that mean different (excited) quarks. The minimum energy of the energy band is the rest mass of the excited quark. Each energy band with a rest mass (m) and the intrinsic quantum numbers (I, S, C, b and Q) corresponds to a quark with the same rest mass and the same intrinsic quantum numbers. All quarks inside baryons and mesons are the excited states of the elementary ϵ-quark.

4. The strange number S of an excited quark that lies on an axis inside the regular rhombic dodecahedron is

$$\text{S} = \text{R - 4}. \tag{6}$$

For the three axes (the P-N axis, the P-H axis and the M-N axis) on the surface of the regular rhombic dodecahedron, the strange numbers are as follows:

$$\begin{array}{l} \text{the P-N axis parallels with the } \Gamma\text{-H axis, } \ S_{\text{P-N}} = S_{\Gamma\text{-H}} = 0; \\ \text{the P-H axis parallels with the } \Gamma\text{-P}'(\frac{1}{2},\ \frac{1}{2},\ \frac{-1}{2}) \text{ axis, } S_{\text{P-H}} = S_{\Gamma\text{-P}'} = \text{- } 1; \\ \text{the M-N axis parallels with the } \Gamma\text{-N}'(\frac{1}{2},\ \frac{-1}{2},\ 0) \text{ axis, } S_{\text{M-N}} = S_{\Gamma\text{-N}'} = \text{- } 2. \end{array} \tag{7}$$

Since the Γ-P$'(\frac{1}{2}, \frac{1}{2}, \frac{-1}{2})$ axis inside the regular rhombic dodecahedron, R = 3, is an equivalent axis (Table A2) of the Γ-P axis, S = -1 . The Γ-N$'$ axis inside the regular rhombic dodecahedron, R = 2, is an equivalent axis (Table A2) of the Γ-N axis, thus S = -2.

5. For any valid value of the integers $\overrightarrow{\text{n}}$ (66) in Appendix A, substituting the (ξ, η, ζ) coordinates (5) of an axis into formula (2), we can get the energy bands that are shown in Table B1–Table B7 (see Appendix B).

From these tables, we can see that there are eightfold, sixfold, fourfold, threefold and twofold energy bands. A group of energy bands (number=deg) with the same energy and equivalent $\overrightarrow{\text{n}}$ values (66) are called the **degenerate energy bands**. For a group of degenerate energy bands (number = deg), its isospin I is determined by

$$\text{deg} = 2\text{I} + 1. \tag{8}$$

The formal z-components of the isospin are as follows:

for I = $\frac{3}{2}$,	$I'_z = \frac{3}{2}, \frac{1}{2}, \frac{-1}{2}, \frac{-3}{2}$;
for I = 1,	I'_z = 1, 0, -1;
for I = $\frac{1}{2}$,	$I'_z = \frac{1}{2}, \frac{-1}{2}$;
for I = 0,	I'_z = 0.

(9)

If a group of g-fold energy bands with the same energy and g > R - the rotary fold of the symmetry axis (see Table A3 in Appendix A)

$$g > R, \tag{10}$$

the g-fold bands will be divided into γ sub-fold energy bands (each has R-bands) first (we call it the first division, K = 0; the energy and the strange numbers are not changed in the first division),

$$\gamma = g/R. \tag{11}$$

If the sub-fold energy bands are degenerate, using (8), we can find the isospin value for the sub-fold degenerate energy bands. Since each sub-degeneracy group has R-fold degenerate bands

$$I = (R\text{-}1)/2. \tag{12}$$

For sub-fold energy bands with non-equivalent n values, the sub-fold energy bands will be divided into sub-subgroups with equivalent (or single) $\overrightarrow{n}$ values (the second kind of division, K = 1) again (see Table A3). Then, using (8), we can find the isospin values of the sub-subgroups.

6. From (6) and (8), the single energy bands on the Γ-H axis (see Table B1) will have I = S = 0 and the single energy bands on the Γ-N axis (see Table B2) will have I = 0 and S = -2. Since there is not any quark that has I = 0 and S = 0 or S = -2 in

the current Quark Model, we need a new formula to deduce the correct S values for the single bands.

For the single energy bands on the Γ-H and the Γ-N axes, the strange number

$$S = S_{axis} + \Delta S \tag{13}$$

$$\Delta S = \delta(\widetilde{n}) + [1\text{-}2\delta(S_{axis})]\mathrm{Sign}(\widetilde{n}), \tag{14}$$

where $\delta(\widetilde{n})$ and $\delta(S_{axis})$ are Dirac functions, and S_{axis} is the strange number of the axis (see Table A3). For an energy band with $\overrightarrow{n}$ = (n_1, n_2, n_3), $\widetilde{n}$ is defined as

$$\widetilde{n} \equiv \frac{n_1+n_2+n_3}{|n_1|+|n_2|+|n_3|}. \tag{15}$$

$$\mathrm{Sign}(\widetilde{n}) = \begin{bmatrix} +1 \text{ for } \widetilde{n} > 0 \\ 0 \quad \text{for } \widetilde{n} = 0 \\ \text{-1} \quad \text{for } \widetilde{n} < 0 \end{bmatrix}. \tag{16}$$

$$\text{If } \widetilde{n} = 0 \qquad \Delta S = \delta(0) = +1. \tag{17}$$

If $\quad \widetilde{n} = \frac{0}{0}$,

$$\Delta S = \text{-}\ S_{Axis}. \tag{18}$$

Thus, for $\overrightarrow{n}$ = (0, 0, 0), from (15) and (18), we have

$$\mathrm{Sign}(\widetilde{n}) = \frac{n_1+n_2+n_3}{|n_1|+|n_2|+|n_3|} = \frac{0}{0}.$$

$$S = S_{Axis}+\Delta S = S_{Axis}\text{-}\ S_{Axis} = 0. \tag{19}$$

Table 2. The S-Number of the Bands with $\overrightarrow{n}$ = (0, 0, 0)

$\overrightarrow{n}$	Axis	Energy Band	S_{Axis}	ΔS	S	$\Delta E^{\#}$
(0, 0, 0)	Δ	E_Γ=0→E_H=1	0	0	0	0
(0, 0, 0)	Λ	E_Γ=0→$E_P=\frac{3}{4}$	-1	+1	0	0
(0, 0, 0)	Σ	E_Γ=0→$E_N=\frac{1}{2}$	-2	+2	0	0
(0, 0, 0)	D	$E_N=\frac{1}{2}$ →$E_P=\frac{3}{4}$	0	0	0	0
(0, 0, 0)	F	$E_P=\frac{3}{4}$ →E_H=1	-1	+1	0	0
(0, 0, 0)	G	$E_N=\frac{1}{2}$ →E_M=1	-2	+2	0	0

\# Since S = C = b =0, from (30), ΔE = 0

7. We believe that the ΔS is the result of the fluctuation of strange numbers of excited energy bands. Some energy bands have the fluctuations of the strange numbers (ΔS ≠ 0), while some energy bands do not have the fluctuation of strange numbers (ΔS = 0). It is necessary to find the conditions that determine the fluctuation of the strange numbers.

For the R-degenerate bands of an axis with rotary fold R, $\Delta S = 0$. Such as: the fourfold bands of the Δ-axis, the threefold bands of the Λ-axis and the twofold degenerate bands of the D-axis, Σ-axis and G-axis. For a group of energy bands with fold = γR (γ is an integer number 2 or 3 or 4), after first division, there may be γ (R-degenerate) energy bands that have ΔS = 0. For a group of degenerate energy bands with the deg < R, using (6), we can get S; with (8), we can get I. If it has the same S and I as a quark of the current Quark Model, ΔS = 0. For example, for the single energy bands of the Λ-axis (or the F-axis), it has S = -1 and I = 0 that is the same as the s-quark has, ΔS = 0. The results are shown in Table A4 of Appendix A.

Otherwise, ΔS ≠ 0, for a group with the deg < R that has S and I that no quark of the current Quark Model has. For the single bands of the Δ-axis, Σ-axis, D-axis and G-axis, and the double degenerate bands of the F-axis, ΔS ≠ 0. Detailed results are shown in Table A5.

8. The “strange number” from (13) is not exactly the same as the strange number from (6). In order to compare them with the experimental results, we would like to give new names under certain circumstances. The new names will be the charmed number and the bottom number. If S = +1, we call it the charmed number C:

$$\text{if } \Delta\text{S} = +1 \rightarrow \text{S} = \text{S}_{Ax} + \Delta\text{S} = +1,\ C \ \equiv\ +1. \tag{20}$$

If S = -1, which originates from ΔS = +1 on a single energy band, and there is an energy fluctuation, we call it the bottom number b:

$$\text{for single bands, if } \Delta\text{S} = +1 \rightarrow \text{S} = \text{-1 and } \Delta\text{E} \neq 0,\ \text{b} \ \equiv\ \text{-1}. \tag{21}$$

Similarly, we can obtain charmed strange quarks q_{Ξ_C} and q_{Ω_C}.

9. The sixfold energy bands of the F-axis (R = 3) need two divisions. In the first division, the sixfold band divides into two ($\frac{6}{R}$=2) threefold bands; the energy and the strange number do not change (S = -1 steel). In the second division (K = 1), the threefold bands with non-equivalent n values divide into a twofold band and a single band. For the twofold energy band, if

$$[(\text{for sixfold bands})\ \Delta\text{S} = +1 \text{ and } \ \text{E} > \text{m}_{u_c}(1753 \text{ Mev})] \rightarrow \text{q}_{\Xi_C}, \tag{22}$$

the twofold energy band represents a twofold family q_{Ξ_C}-quark with S = -1 and C = +1.

The sixfold energy bands of the G-axis (R = 2) need two divisions also. In the first division, the sixfold band divides into three ($\frac{6}{R}$=3) twofold bands. The energy and the strange number do not change (S = -2). In the second division (K = 1), the twofold band with non-equivalent $\overrightarrow{n}$ values divides into two single bands. For a single energy band, if

$$[(\text{for sixfold bands})\ \Delta\text{S} = +1 \text{ and } \ \text{E} > \text{m}_{u_c}(1753 \text{ Mev})] \rightarrow \text{d}_{\Omega_C}, \tag{23}$$

the single energy band represents a d_{Ω_C}-quark with S = -2 and C = +1.

10. The elementary quark ϵ_u (or ϵ_d) determines the electric charge Q of an excited quark ϵ_u (or ϵ_d). For an excited quark of ϵ_u (or ϵ_d), Q = $+\frac{2}{3}$ (or $-\frac{1}{3}$). For a quark with isospin I , there are 2I +1 members . Since the ϵ_u-quark has $I_z = \frac{1}{2} > 0$ and the ϵ_d-quark has $I_z = -\frac{1}{2} < 0$, for an excited quark with $I_z^{'} > 0$ from (9), it is an excited quark of ϵ_u, its electric charge Q_q is

$$Q_q = Q_{\epsilon_u} = \frac{2}{3}. \tag{24}$$

For an excited quark with $I_z^{'} < 0$ from (9), it is an excited quark of ϵ_d, its electric charge Q_q is

$$Q_q = Q_{\epsilon_d} = -\frac{1}{3}. \tag{25}$$

For $I_z^{'} = 0$, if S + C + b > 0,

$$Q_q = Q_{\epsilon_u} = \frac{2}{3}; \tag{26}$$

if S + C + b < 0,

$$Q_q = Q_{\epsilon_d} = -\frac{1}{3}. \tag{27}$$

There is not any quark with $I_z^{'} = 0$ and S + C + b = 0.

11. After getting S, C, b and $Q_{q_{z'}}$ ($B = \frac{1}{3}$) of a quark, we can deduce a physical I_z of the quark using the generalized Gell-Mann-Nishijima relationship [9]:

$$Q_{q_{z'}} = I_z + \frac{1}{2}(B + S + C + b), \tag{28}$$

where B is the baryon number ($B = \frac{1}{3}$) and $Q_{q_{z'}}$ is the electric charge of the quark. For the d_S-quark, $B = \frac{1}{3}$, S = -1, C = b = 0 and $Q = -\frac{1}{3}$, from (28), $I_Z = 0$; for the u_C-quark, $B = \frac{1}{3}$, C = 1, S = b = 0 and $Q = +\frac{2}{3}$, from (28), $I_Z = 0$. The physical I_Z-values of all low energy quarks will be shown in Table 10.

12. The rest masses m^* of the excited quarks are the minimum energy of the energy band (2),

$$m^* = \{V_0 + \alpha \times \text{minimum}[(n_1-\xi)^2 + (n_2-\eta)^2 + (n_3-\zeta)^2] + \Delta E\}, \tag{29}$$

where ΔE is the fluctuate energy of the excited quark. The fluctuations will increase as the energies increase. In order to take this effect into account, we define a fluctuation order number J. J depends on the symmetry axis, the symmetry point, energy bands and ΔS. If $\Delta S = 0$, $J = 0$. For the energy bands on the Λ-axis, $\Delta S = 0$, $J = 0$. For a symmetry axis, the same fold energy bands with the same ΔS and the same intrinsic quantum numbers (S, C and b), the fluctuation numbers are positive order integers 1, 2, 3, ... from the lowest energy band to higher bands. We give the J values for all fluctuation energy bands sufficient to cover the experimental data in Table B1-Table B7. The fluctuation energy ΔE can be found with a phenomenological formula as follows:

$$\begin{gathered} \Delta E = 100\{\Theta C[2(J_C - 3.5I) - S] + KS + (S + b)[(1 + S_{Ax})(J_{S,b} + S_{Ax})]\Delta S\} \\ J_C = 1, 2, 3, \ldots \quad J_{S,b} = \Theta(1 - S_{Ax}) + (1, 2, 3, \ldots) \;\; (\Delta E = 0, \text{ for } J \leq \Theta(1\text{-}S_{Ax})). \end{gathered} \tag{30}$$

From Table A3, for the axes Δ, Σ and Λ, $\Theta = 0$; for the axes D, F and G, $\Theta = 1$. S_{Ax} is the strange number of the symmetry axis. J_C is the order number of the charmed quarks from low energy to high energy in a symmetry axis (see Table B5, Table B6 and Table B7). $J_{S.b}$ is the fluctuation order number of the fluctuation energy bands with $S \neq 0$ or $b \neq 0$. As an example, we can deduce the ΔE of the energy bands with $\overrightarrow{n} = (0, 0, 0)$ using (30). From Table 2, for these energy bands, $C = S = b = 0$, we have $\Delta E = 0$.

We find the parameters V_0 and α now. According to the Quark Model [1], a baryon is composed of three different colored quarks. In particular the proton and the neutron are given by

$$p = uud \text{ and } n = udd,$$

so that, if E_{bind} represents the binding energy of a proton or a neutron, then

$$\begin{aligned} M_P &= m_u + m_u + m_d - |E_{bind}|\,, \\ M_n &= m_u + m_d + m_d - |E_{bind}|. \end{aligned} \tag{31}$$

If omitting electric masses, we have an approximation,

$$\begin{aligned} &M_P \sim M_n = 939 \text{ Mev}, \\ &m_u = m_d. \end{aligned} \tag{32}$$

Thus, from (31) and (32), we have

$$m_u = m_d = \frac{1}{3}(939 + |E_{bind}|\,) = 313 + \frac{1}{3}\,|E_{bind}|\,, \tag{33}$$

where E_{bind} is the total binding energy of the three quarks (colors) in a baryon. We use $\Delta = \frac{1}{3}\,|E_{bind}|$ that is the phenomenological approximations of the color's strong interaction energies. Since the three colors are the same for all baryons, Δ is an unknown positive constant for all baryons. It originates from the three colors of the three quark inside the baryons. Since high energy scattering experiments (an uncounted number) have not separated the quarks inside the baryons, it means that $|E_{bind}|$ (3Δ) is much larger than M_p,

$$\Delta = \frac{1}{3}\,|E_{bind}| \;>>\; M_p. \tag{34}$$

The lowest mass of the quarks is the rest mass of the u-quark (or the d-quark) (33). From (29), (33) and (34), the lowest quark mass is

$$m^*_{\text{Lowest}} = V_0 = (313+\Delta) \text{ Mev}. \tag{35}$$

Fitting experimental results, we can get

$$\alpha = 360 \text{ Mev}. \tag{36}$$

The rest mass (m^*) of a quark, from (29), is

$$\begin{aligned} m^* &= \{313 + 360 \text{ minimum}[(n_1\text{-}\xi)^2 + (n_2\text{-}\eta)^2 + (n_3\text{-}\zeta)^2] + \Delta E + \Delta\} \text{ (Mev)} \\ &= m + \Delta \;\; \text{(Mev)}, \end{aligned} \tag{37}$$

where ΔE is in (30). Δ is an unknown large constant ($\Delta >> M_P$). Since (-3Δ) is the three colors interaction energy, from (33) and (34), the Δ part in (37) of quark mass is from the color. This formula (37) is the united rest mass formula for quarks.

In the next section, we will use the above phenomenological formulae to deduce the rest masses and the intrinsic quantum numbers (S, C, b, I and Q) of the quarks (a quark spectrum).

III The Quark Spectrum

Using the above formulae, we can deduce an excited (from the vacuum) quark spectrum. We will find the energy band excited states of the elementary ϵ-quarks first.

A The Energy Bands

In order to show how to calculate the energy bands, we give the calculation of some low energy bands in the Δ-axis as an example.

First we find the formulae for the E($\vec{k}$, $\vec{n}$) at the points Γ, H and Δ of the Δ-axis (see Fig. 1). From (2), (4), (35) and (36), we get:

$$E(\vec{k},\vec{n}) = 313+\Delta+ 360\ E_{\vec{k}}(n_1, n_2, n_3); \tag{38}$$

$$E_{\vec{k}}(n_1, n_2, n_3) = [(n_1-\xi)^2+(n_2-\eta)^2+(n_3-\zeta)^2].$$

$$\text{For } \Gamma = (0,\ 0,\ 0),\ E(\vec{k},\vec{n}) = 313+\Delta+ 360E_{\Gamma}, \quad E_{\Gamma} = (n_1^2 + n_2^2 + n_3^2); \tag{39}$$

$$\text{for } H = (0,\ 0,\ 1),\ E(\vec{k},\vec{n}) = 313+\Delta+ 360E_{H}, \quad E_{H} = [n_1^2 + n_2^2 + (n_3-1)^2]; \tag{40}$$

for $\Delta = (0, 0, \zeta)$, $E(\vec{k},\vec{n}) = 313+\Delta+360E_\Delta$, $\quad E_\Delta = [n_1^2 + n_2^2 + (n_3-\zeta)^2]$. (41)

Then, using (39)–(41) and beginning from the lowest energy, we get:

A1. The lowest $E(\vec{k}, \vec{n})$ is at $(\xi, \eta, \zeta) = 0$ (the Γ-point) and $\vec{n} = (0, 0, 0)$. From (39) and (38) , we have

$$\vec{n} = (0, 0, 0),\ E_\Gamma(0, 0, 0) = 0,\ E(\overrightarrow{0}, \overrightarrow{0}) = 313 + \Delta. \tag{42}$$

A2. Starting from $E_\Gamma = 0$ [$E(\overrightarrow{0}, \overrightarrow{0}) = 313 + \Delta$ (Mev)], from (35) and (36), we find that there is one energy band (the lowest energy band) $E_\Delta = \zeta^2$ ($\zeta = 0 \to 1$) along the Δ-axis, with $n_1 = n_2 = n_3 = 0$ (see(41)) ended at the point $E_H = 673 + \Delta$:

$$\vec{n} = (0, 0, 0) \text{ (single band)}, \tag{43}$$

$$E_\Gamma = 0 \ \to\ E_\Delta = \zeta^2\ (\zeta = 0 \to 1) \to E_H = 1, \tag{44}$$

$$E(\Gamma,0) = 313+\Delta \to E(\Delta, 0) = 313+\Delta+\zeta^2 \to E(H,0) = 673+\Delta. \tag{45}$$

A3. At the end point H, the energy $E(H,0) = 673 + \Delta$. When $n = (\pm 1, 0, 1)$, $(0, \pm 1, 1)$, and (0,0,2), $E(H, \overrightarrow{n}) = 673 + \Delta$ also (see (40)). Starting from $E_H = 673 + \Delta$, along the Δ-axis, we find there are three energy bands ending at the points $E_\Gamma = 313 + \Delta$, $E_\Gamma = 1033 + \Delta$ and $E_\Gamma = 1753 + \Delta$, respectively:

$\vec{n} = (0, 0, 0)$ (single band),

$E_H = 1 \to E_\Delta = \zeta^2$ ($\zeta = 1 \to 0) \to E_\Gamma = 0$,

$E(H, 0) = 673+\Delta \to E(\Delta, 0) = 313+\Delta+\alpha\ \zeta^2 \to E(\Gamma, 0) = 313+\Delta$;

$\vec{n} = (0, 0, 2)$ (single band),

$E_H = 1 \to E_\Delta = (2-\zeta)^2$ ($\zeta = 1 \to 0) \to E_\Gamma = 4$,

$E(H,002) = 673+\Delta \to E_\Delta = 313+\Delta+\alpha(2-\zeta)^2 \to E(\Gamma, 002) = 1753+\Delta$;

$\vec{n}$ = (±1, 0, 1) and (0, ± 1, 1) (fourfold degeneracy),

E_H =1 → E_Δ= 313+Δ + $\alpha[1+(1-\zeta)^2]$ (ζ=1 → 0) → E_Γ = 2,

E(H;±1,0,1) =673+Δ → E_Δ=673+Δ+$\alpha(1-\zeta)^2$ → E(Γ;±1,0,1)=1033+Δ,

E(H;0, ±1,1) =673+Δ → E_Δ=673+Δ+$\alpha(1-\zeta)^2$ → E(Γ;0,±1,1)=1033+Δ.

Continuing this process, we can find all low energy bands of the Δ-axis. We show the energy bands in Table B1 of the Appendix B.

Similarly, we deduce all low energy bands on the Λ-axis, the Σ-axis, the D-axis, the F-axis and the G-axis. These low energy bands are sufficient to cover experimental data. We show these energy bands in Table B2–Table B7 of the Appendix B.

B The Excited Quarks of the Elementary ϵ-Quarks

B- 1 The Quarks on the Δ-Axis (the Γ-H axis)

Since the Δ-axis is a fourfold rotatory axis (see Fig. 1), R = 4. From (6), we get S = 0. Because the axis has $R = 4$, we can use (10) and (11) to determine that the energy bands with 8-fold degeneracy will be divided into two fourfold degenerate bands (K = 0 from Table A3).

1. The Quarks of the Fourfold Degenerate Bands on the Δ-Axis

For fourfold degenerate bands, using (8), we get I = $\frac{3}{2}$ and I'_Z = $\frac{3}{2}$, $\frac{1}{2}$, -$\frac{1}{2}$, -$\frac{3}{2}$, from (9). Thus each fourfold degenerate band represents a fourfold quark family, q_Δ(=$q_\Delta^{\frac{3}{2}}$, $q_\Delta^{\frac{1}{2}}$, $q_\Delta^{\frac{-1}{2}}$, $q_\Delta^{\frac{-3}{2}}$), with

$$B = \frac{1}{3}, S = 0, I = \frac{3}{2}, I'_z = \frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}. \qquad (46)$$

Using Table B1, we can get E_Γ, E_H and $\vec{n}$ values. When we put the values of E_Γ and E_H into the rest mass formula (37), we can find the rest mass $m^*_{q_\Delta}$. Since Θ = K = ΔS = 0 {see Table A3 and A4}, from (30), ΔE =0. Thus we have [q_Δ = ($q_\Delta^{\frac{3}{2}}$, $q_\Delta^{\frac{1}{2}}$, $q_\Delta^{\frac{-1}{2}}$, $q_\Delta^{\frac{-3}{2}}$)]:

Table 3. The $q_\Delta(m^*)$-quarks

E_{Point}	$(n_1 n_2 n_3, \ldots)$	$E(\vec{k}, \vec{n})$	I	ΔS	J	ΔE	$q_\Delta(m^*(Mev))$
E_H=1	(101,-101,011,0-11)	673	$\frac{3}{2}$	0	0	0	$q_\Delta(673+\Delta)$
E_Γ=2	(110,1-10,-110,-1-10,	1033	$\frac{3}{2}$	0	0	0	$q_\Delta(1033+\Delta)$
	10-1,-10-1,01-1,0-1-1)	1033	$\frac{3}{2}$	0	0	0	$q_\Delta(1033+\Delta)$
E_H=3	(112,1-12,-112,-1-12)	1393	$\frac{3}{2}$	0	0	0	$q_\Delta(1393+\Delta)$
E_Γ=4	(200,-200,020,0-20)	1753	$\frac{3}{2}$	0	0	0	$q_\Delta(1753+\Delta)$
E_H=5	(121,1-21,-121,–1-21,	2113	$\frac{3}{2}$	0	0	0	$q_\Delta(2113+\Delta)$
	211,2-11,-211,-2-11)	2113	$\frac{3}{2}$	0	0	0	$q_\Delta(2113+\Delta)$
E_H=5	(202,-202,022,0-22)	2113	$\frac{3}{2}$	0	0	0	$q_\Delta(2113+\Delta)$
E_H=5	(013,0-13,103,-103)	2113	$\frac{3}{2}$	0	0	0	$q_\Delta(2113+\Delta)$
E_Γ=6	$(12\bar{1},1\bar{2}\bar{1},\bar{1}21,\bar{1}\bar{2}\bar{1},$	2473	$\frac{3}{2}$	0	0	0	$q_\Delta(2473+\Delta)$
	$21\bar{1},2\bar{1}\bar{1},\bar{2}1\bar{1},\bar{2}\bar{1}\bar{1})$	2473	$\frac{3}{2}$	0	0	0	$q_\Delta(2473+\Delta)$
E_Γ=6	$(11\bar{2},1\bar{1}\bar{2},\bar{1}1\bar{2},\bar{1}\bar{1}\bar{2})$	2473	$\frac{3}{2}$	0	0	0	$q_\Delta(2473+\Delta)$

2. The Quarks of the Single Energy Bands on the Δ-Axis

For the single bands on the Δ-axis (see Table B1), R = 4, S_Δ = 0 from (6); deg = 1, I = 0 from (8). For the single bands, using (13) instead of (6), we find that ΔS = +1 at E_Γ = 4, 16, 36, ... from (14) and ΔS = -1 at E_H= 1, 9, 25, 49, ... from (14). For $\vec{n}$ = (0, 0, 0), from Table 2, S = C = b = 0; from (30), ΔE = 0. For other energy bands, Θ = K = b = 0 (see Table A3), from (30), ΔE = 100 S $J_S \Delta S$, J_S = 1, 2, 3,.... Using (20), we have:

Table 4. The u_C(m^*)-quarks and the d_S(m^*)-quarks

E_{Point}	$E(\overrightarrow{k},\overrightarrow{n})$	n_1,n_2,n_3	ΔS	J	I	S	C	ΔE	q_{Name}(m^*(Mev))
E_Γ=0	313	0, 0, 0	0	J = 0	$\frac{1}{2}$	0	0	0	u(313+Δ)
E_H=1	673	0, 0, 2	-1	$J_{S,H}$=1	0	-1	0	100	d_S(773+Δ)
E_Γ=4	1753	0, 0, -2	+1	$J_{C,\Gamma}$=1	0	0	1	0	u_C(1753+Δ)
E_H=9	3553	0, 0, 4	-1	$J_{S,H}$=2	0	-1	0	200	d_S(3753+Δ)
E_Γ=16	6073	0, 0, -4	+1	$J_{C,\Gamma}$=2	0	0	1	0	u_C(6073+Δ)
E_H=25	9313	0, 0, 6	-1	$J_{S,H}$=3	0	-1	0	300	d_S(9613+Δ)
E_Γ=36	13273	0, 0, -6	+1	$J_{C,\Gamma}$=3	0	0	1	0	u_C(13273+Δ)
E_H=49	17953	0, 0, 8	-1	$J_{S,H}$=4	0	-1	0	400	d_S(18353+Δ)

B- 2 The Quarks on the Σ-Axis (Γ-N)

The Σ-axis is a twofold rotatory axis, R = 2, from (6) S = - 2 .

1. The Quarks of the Twofold Degenerate Energy Bands on the Σ-axis (Γ-N)

For the twofold degenerate energy bands, each represents a quark family q_Ξ ($q_\Xi^{\frac{1}{2}}$, $q_\Xi^{-\frac{1}{2}}$) with B = 1/3, S = -2, I = 1/2 from (8), $I_z' = \frac{1}{2}$, -$\frac{1}{2}$ from (9) and Q =$\frac{2}{3}$, $\frac{-1}{3}$ from (24) and (25). Since Θ = K = ΔS = 0 (see Table A3 and Table A4), from (30), ΔE =0. Similar to Table 3, we have [q_Ξ ($q_\Xi^{\frac{1}{2}}$, $q_\Xi^{-\frac{1}{2}}$)]:

Table 5. The q_Ξ(m^*)-quarks on Twofold Energy Bands

E_{Point}	$n_1n_2n_3$	S_Ξ	I	$E(\overrightarrow{k},\overrightarrow{n})$	ΔS	J	ΔE	q_Ξ(m^*(Mev))
$E_\Gamma = 2$	(1-10,-110)	-2	$\frac{1}{2}$	1033	0	0	0	q_Ξ(1033+Δ)
$E_N = \frac{5}{2}$	(200,020)	-2	$\frac{1}{2}$	1213	0	0	0	q_Ξ(1213+Δ)
$E_\Gamma = 4$	(002,00-2	-2	$\frac{1}{2}$	1753	0	0	0	q_Ξ(1753+Δ)
	-200,0-20)	-2	$\frac{1}{2}$	1753	0	0	0	q_Ξ(1753+Δ)
$E_N = \frac{9}{2}$	(112,11-2)	-2	$\frac{1}{2}$	1933	0	0	0	q_Ξ(1933+Δ)
...	...	...	...	...	...	...	...	....

2. The Quarks of the Fourfold Degenerate Energy Bands on the Σ−Axis

According to (10) and (11), each fourfold degenerate energy band on the Σ-axis with R = 2 divides into two twofold degenerate bands. From Table 5, each of them represents a quark family q_Ξ ($q_\Xi^{\frac{1}{2}}$, $q_\Xi^{-\frac{1}{2}}$) with B = 1/3, S = -2, I = 1/2, $I_z' = \frac{1}{2}$, -$\frac{1}{2}$ and Q = $\frac{2}{3}$, $\frac{-1}{3}$.

Thus we have [since Θ = K = ΔS = 0, from (30), ΔE =0]:

Table 6. The $q_{\Xi}(m^*)$-quarks on Fourfold Energy Bands

E_{Point}	($n_1n_2n_3$, ...)	I	S	ΔS	J	ΔE	$q_{\Xi}(m^*(Mev))$
$E_N = \frac{3}{2}$	(101,10-1,011,01-1)	$\frac{1}{2}$	-2	0	0	0	$2 \times q_{\Xi}(853+\Delta)$
$E_\Gamma = 2$	(-101,-10-1,0-11,0-1-1)	$\frac{1}{2}$	-2	0	0	0	$2 \times q_{\Xi}(1033+\Delta)$
$E_N = \frac{7}{2}$	(121,12-1,211,21-1)	$\frac{1}{2}$	-2	0	0	0	$2 \times q_{\Xi}(1573+\Delta$
...	...	...	...	...	...	...	... .

3. The Quarks of the Single Energy Bands on the Σ-axis (Γ-N)

For the single bands on the Σ-axis (see Table B2), R = 2, S_Σ = -2 from (6); deg = 1, I = 0 from (8). Using (13) instead of (6), we get ΔS = +1 at $E_N = \frac{1}{2}, \frac{9}{2}, \frac{25}{2}, \frac{49}{2}, \frac{81}{2}$, ... from (14); at E_Γ = 2, 8, 18, 32, ... ΔS = -1 from (14) . For $\overrightarrow{n}$ = (0, 0, 0), from Table 2, S = b = Θ = K = 0, ΔE = 0 (30). For other bands, since Θ = K = 0, from (30), ΔE = - 100(S+b)($J_{S,b}$-2)ΔS, J_S = 3, 4, 5, ...; ΔE = 0 $J_S < 3$. Using (21), similar to Table 4, we have:

Table 7. The $d_\Omega(m^*)$-quarks and $d_S(m^*)$-quarks

E_{Point}	$n_1n_2n_3$	ΔS	S	b	J	I	$E(\overrightarrow{k},\overrightarrow{n})$	ΔE	$d(m^*(Mev))$
E_Γ=0	(0, 0, 0)	+2	0	0	$J_{S,\Gamma}$ =0	$\frac{1}{2}$	313	0	d(313+Δ)
$E_N=\frac{1}{2}$	(1,1,0)	+1	-1	0	$J_{S,N}$ =1	0	493	0	d_S(493+Δ)
E_Γ=2	(-1,-1,0)	-1	-3	0	$J_{S,\Gamma}$ =1	0	1033	0	d_Ω(1033+Δ)
$E_N=\frac{9}{2}$	(2,2,0)	+1	-1	0	$J_{S,N}$ =2	0	1933	0	d_S(1933+Δ)
E_Γ=8	(-2,-2,0)	-1	-3	0	$J_{S,\Gamma}$ =2	0	3193	0	d_Ω(3193+Δ)
$E_N=\frac{25}{2}$	(3,3,0)	+1	0	-1	$J_{S,N}$ =3	0	4813	100	d_b(4913+Δ)
E_Γ=18	(-3,-3,0)	-1	-3	0	$J_{S,\Gamma}$ =3	0	6793	-300	d_Ω(6493+Δ)
$E_N=\frac{49}{2}$	(4,4,0)	+1	0	-1	$J_{S,N}$ =4	0	9133	200	d_b(9333+Δ)
E_Γ=32	(-4,-4,0)	-1	-3	0	$J_{S,\Gamma}$ =4	0	11833	-600	d_Ω(11233+Δ)
$E_N=\frac{81}{2}$	(5,5,0)	+1	0	-1	$J_{S,N}$ =5	0	14893	300	d_b(15193+Δ)

B- 3 The Quarks on the Λ-Axis (Γ-P)

Since the Λ-axis is a threefold rotatory axis (see Fig. 1), R = 3, from (6) we have S = -1. From Table B3, we see that there are two single energy bands with $\vec{n}$ = (0, 0, 0) and $\vec{n}$ = (2, 2, 2), and all other bands are either threefold degenerate energy bands (deg = 3) or sixfold degenerate bands (deg = 6). From (10) and (11), the sixfold degenerate energy bands will divide into two threefold energy bands.

1. The Quarks of the Threefold Degenerate Energy Bands on the Λ-Axis (Γ-P)

For the threefold degenerate energy bands, using (8), (9), (24) and (25), we have I = 1 and I_z' = 1, 0, -1. Thus we get a three-member quark family $q_\Sigma(q_\Sigma^1, q_\Sigma^0, q_\Sigma^{-1})$ with B = 1/3, S = -1 and I = 1. Since Θ = K = ΔS = 0, from (30), ΔE =0. Using Table B3, we have:

Table 8. The $q_\Sigma(m^*)$-quarks (S = S_{Ax} + ΔS = -1)

E_{Point}	($n_1n_2n_3$, ...)	$E(\overrightarrow{k},\overrightarrow{n})$	I	ΔS	J	ΔE	$q_\Sigma(m^*)$
$E_P=\frac{3}{4}$	(101,011,110)	583	1	0	0	0	$q_\Sigma(583+\Delta)$
E_Γ=2	(1-10,-110,01-1,	1033	1	0	0	0	$q_\Sigma(1033+\Delta)$
	0-11,10-1,-101)	1033	1	0	0	0	$q_\Sigma(1033+\Delta)$
E_Γ=2	(-10-1,0-1-1,-1-10)	1033	1	0	0	0	$q_\Sigma(1033+\Delta)$
$E_P=\frac{11}{4}$	(020,002,200)	1303	1	0	0	0	$q_\Sigma(1303+\Delta)$
$E_P=\frac{11}{4}$	(121,211,112)	1303	1	0	0	0	$q_\Sigma(1303+\Delta)$
E_Γ=4	(0-20,-200,00-2)	1753	1	0	0	0	$q_\Sigma(1753+\Delta)$
$E_P=\frac{19}{4}$	(1-12,-112,21-1,	2023	1	0	0	0	$q_\Sigma(2023+\Delta)$
	2-11,12-1,-121)	2023	1	0	0	0	$q_\Sigma(2023+\Delta)$
$E_P=\frac{19}{4}$	(202,022,220)	2023	1	0	0	0	$q_\Sigma(2023+\Delta)$
E_Γ=6	(-211,2-1-1,-1-12,	2473	1	0	0	0	$q_\Sigma(2473+\Delta)$
	11-2,-12-1,1-21)	2473	1	0	0	0	$q_\Sigma(2473+\Delta)$
E_Γ=6	(-1-21,1-2-1,-11-2,	2473	1	0	0	0	$q_\Sigma(2473+\Delta)$
	1-1-2,-21-1,-2-11)	2473	1	0	0	0	$q_\Sigma(2473+\Delta)$
E_Γ=6	(-1-2-1,-1-1-2,-2-1-1)	2473	1	0	0	0	$q_\Sigma(2473+\Delta)$
...	...	...	...	...	...	...	....

2. The Single Energy Bands

For the single bands, S_{Ax} = -1, I = 0 and ΔE = 0. From Table 2, for $\overrightarrow{n}$= (0, 0, 0), ΔS = -S_{axis} $\rightarrow$ S = 0. At E_P = $\frac{27}{4}$, $\overrightarrow{n}$= (2, 2, 2), S_{Ax} = -1, I = 0 and ΔS = ΔE = 0:

Table 9. The $d_S(m^*)$-quarks on the Single Bands of the Λ-axis

E_{Point}	n_1, n_2, n_3	$E(\overrightarrow{k}, \overrightarrow{n})$	I	S_{Axis}	ΔS	J	S	C	b	d(m*(Mev))
E_Γ= 0	(0, 0, 0)	313	$\frac{1}{2}$	-1	1	0	0	0	0	d(313+Δ)
$E_P = \frac{27}{4}$	(2, 2, 2)	2743	0	-1	0	0	-1	0	0	d_S(2743+Δ)
...	...	...	...	...		...	...	...	...	... .

Continuing the above procedure, we can deduce low energy excited quarks on the D-axis, the F-axis and the G-axis. Using Table B1-Table B7, we show the excited quarks of low energies that are sufficient to cover experimental data (see Appendix B).

C The Quark Spectrum

In this section, from the energy bands in Table B1-Table B7, using phenomenological formulae S = R - 4, deg = 2I + 1, S = S_{Ax}+ΔS {ΔS = $\delta(\widetilde{n})$ + [1-2$\delta(S_{axis})$]Sign($\widetilde{n}$)}, (24), (25), (20), (21), (22) and (23), we have deduced the strange numbers (S), the isospins (I), the electric charges (Q), the charmed numbers (C) and the bottom numbers (b) of the quarks. For each deduced quark there are always three different colored members. These three different colored quarks have exactly the same rest masses and intrinsic quantum numbers. We can omit the colors as we show the deduced intrinsic quantum numbers in Table 10

Table 10. The Quantum Numbers of the Quarks

$q^{I'_Z}_{\text{Name}}$	$q^{\frac{1}{2}}_N$	$q^{\frac{-1}{2}}_N$	$q^{\frac{3}{2}}_\Delta$	$q^{\frac{1}{2}}_\Delta$	$q^{\frac{-1}{2}}_\Delta$	$q^{\frac{-3}{2}}_\Delta$	q^{1}_Σ	q^{0}_Σ	q^{-1}_Σ
S	0	0	0	0	0	0	-1	-1	-1
C	0	0	0	0	0	0	0	0	0
b	0	0	0	0	0	0	0	0	0
I	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	1	1	1
I'_Z	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{3}{2}$	1	0	-1
Q_q	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{-1}{3}$	$\frac{-1}{3}$	$\frac{2}{3}$	$\frac{-1}{3}$	$\frac{-1}{3}$
$\epsilon^{I_Z}_{\text{Name}}$	$u^{\frac{1}{2}}_N$	$d^{\frac{-1}{2}}_N$	$u^{\frac{1}{2}}_\Delta$	$u^{\frac{1}{2}}_\Delta$	$d^{\frac{-1}{2}}_\Delta$	$d^{\frac{-1}{2}}_\Delta$	u^{1}_Σ	d^{0}_Σ	d^{0}_Σ
I_Z	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	1	0	0
*****	***	***	***	***	***	***	***	***	***
$q^{I'_Z}_{\text{Name}}$	$q^{\frac{1}{2}}_\Xi$	$q^{\frac{-1}{2}}_\Xi$	q^{0}_S	q^{0}_Ω	q^{0}_C	q^{0}_b	$q^{0}_{\Omega_C}$	$q^{\frac{1}{2}}_{\Xi_C}$	$q^{\frac{-1}{2}}_{\Xi_C}$
S	-2	-2	-1	-3	0	0	-2	-1	-1
C	0	0	0	0	1	0	1	1	1
b	0	0	0	0	0	-1	0	0	0
I	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$
I'_Z	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	0	0	$\frac{1}{2}$	$-\frac{1}{2}$
Q_q	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$
$\epsilon^{I_Z}_{\text{Name}}$	$u^{\frac{3}{2}}_\Xi$	$d^{\frac{1}{2}}_\Xi$	d^{0}_S	d^{1}_Ω	u^{0}_C	d^{0}_b	$d^{0}_{\Omega_C}$	$u^{\frac{1}{2}}_{\Xi_C}$	$d^{\frac{-1}{2}}_{\Xi_C}$
I_z	$\frac{3}{2}$	$\frac{1}{2}$	0	1	0	0	0	$\frac{1}{2}$	$-\frac{1}{2}$

The name of $q^{I'_z}_{\text{Name}}$ is the name of the excited quark; I'_Z is the formal z-component of the isospin of the $q^{I'_z}_{\text{Name}}$ (9). The I_z of $\epsilon^{I_Z}_{\text{Name}}$ is the physical z-component of the isospin of the excited quark $\epsilon^{I_Z}_{\text{Name}}$ from $I_z = Q_q - \frac{1}{2}(B+S+C+b)$. The Q_q is the electric charge of the excited quark $q^{I'_Z}_{\text{Name}}$.

Using the phenomenological united rest mass formula (37), we have deduced the rest masses of the quarks. We show the low rest masses quarks that are sufficient to cover experimental data in Table 11. Since the three different colored quarks with the same flavor have completely the same rest mass and quantum numbers, we can omit the colors in Table 11.

Table 11. The Quark Spectrum
The Elementary Quark ϵ [ϵ_u and ϵ_d]
ϵ_u: I $=\frac{1}{2}$, s $=\frac{1}{2}$, S = C = b =0, $I_z=\frac{1}{2}$, $m_{\epsilon_u(0)}$ = 0 and one of 3 colors
ϵ_d: I $=\frac{1}{2}$, s $=\frac{1}{2}$, S = C = b =0, $I_z=\frac{-1}{2}$, $m_{\epsilon_d(0)}$ = 0 and one of 3 colors
The Excited Quarks $q^{\Delta S}_{\rm Name}$((m+Δ)(Mev)) of ϵ
1. Unflavored Quarks [The Ground Quarks are $\mathbf{u^0(313+\Delta)}$ and $\mathbf{d^0(313+\Delta)}$]
D-Axis: $\mathbf{u^0(313+\Delta)}$, $q^0_{\rm N}(583+\Delta)$, $q^0_{\rm N}(853+\Delta)$,$2q^0_{\rm N}(1213+\Delta)$,$2q^0_{\rm N}(1303+\Delta)$,
$2q^0_{\rm N}(1573+\Delta)$,$3q^0_{\rm N}(2023+\Delta)$, $q^0_{\rm N}(2293+\Delta)$, $2q^0_{\rm N}(2653+\Delta)$.
F-Axis: $\mathbf{d^0(313+\Delta)}$, $q^1_{\rm N}(583+\Delta)$, $q^1_{\rm N}(673+\Delta)$,$3q^1_{N}(1303+\Delta)$,$q^1_{\rm N}(1393+\Delta)$,
$q^1_{\rm N}(2023+\Delta)$.
Δ-Axis: $\mathbf{u^0(313+\Delta)}$, $q^0_{\Delta}(673+\Delta)$, $2q^0_{\Delta}(1033+\Delta)$, $q^0_{\Delta}(1393+\Delta)$, $q^0_{\Delta}(1753+\Delta)$,
$4q^0_{\Delta}(2113+\Delta)$, $3q^0_{\Delta}(2473+\Delta)$.
Λ and F-Axes : $\mathbf{d^0(313+\Delta)}$ ($d^1_S(313+\Delta)$); Σ and G-Axes $\mathbf{d^0(313+\Delta)}$
2. Strange Quarks [The Ground Quark is $\mathbf{d^1_S(493+\Delta)_\Sigma}$]
G-Axis: $2d^1_S(493+\Delta)$, $d^1_S(773+\Delta)$, $d^1_S(1413+\Delta)$, $d^1_S(1513+\Delta)$, $d^1_S(2513+\Delta)$.
Δ-Axis: $d^{-1}_S(773+\Delta)$, $d^{-1}_S(3753+\Delta)$, $d^{-1}_S(9613+\Delta)$, $d^{-1}_S(18353+\Delta)$.
F-Axis: $d^0_S(493+\Delta)$, $d^0_S(773+\Delta)$, $2d^0_S(1203+\Delta)$, $d^0_S(1303+\Delta)$, $2d^0_S(1393+\Delta)$,
$2d^0_{\rm S}(1923+\Delta)$, $4d^0_S(2013+\Delta)$, $d^0_S(2023+\Delta)$, $d^0_S(2643+\Delta)$;
D-Axis: $d^{-1}_{\rm S}(493+\Delta)$, $d^{-1}_{\rm S}(1503+\Delta)$, $d^{-1}_{\rm S}(1603+\Delta)$, $d^{-1}_S(2333+\Delta)$.
Λ-Axis: $d^0_S(2743+\Delta)$, $d^0_{\rm S}(4633+\Delta)$; $q_\Sigma(583+\Delta)$, $3q_\Sigma(1033+\Delta)$, $2q_\Sigma(1303+\Delta)$,
,$q_\Sigma(1753+\Delta)$, $3q_\Sigma(2023+\Delta)$, $5q_\Sigma(2473+\Delta)$, $2q_\Sigma(2743+\Delta)$.
Σ-Axis: $\mathbf{d^1_S(493+\Delta)_\Sigma}$, $d^1_S(1933)$; $2q^0_\Xi(853+\Delta)$, $3q^0_\Xi(1033+\Delta)$, $q^0_\Xi(1213+\Delta)$, .
$2q^0_\Xi(1573+\Delta)$, $2q^0_\Xi(1753+\Delta)$, $q^0_\Xi(1933+\Delta)$, $2q^0_\Xi(2293+\Delta)$.
G-Axis: $q^0_\Xi(673+\Delta)$, $q^0_\Xi(853+\Delta)$, $3q^0_\Xi(1393+\Delta)$, $2q^0_\Xi(1573+\Delta)$, $2q^0_\Xi(1733+\Delta)$,
F-Axis: $q^{-1}_\Xi(1823+\Delta)$, $2q^{-1}_\Xi(1913+\Delta)$. G: $2q^{-1}_\Xi(1913+\Delta)$, $2q^{-1}_\Xi(2113+\Delta)$,
Σ-Axis: $d^{-1}_\Omega(1033+\Delta)$, $d^{-1}_\Omega(3193+\Delta)$; G: $d^{-1}_\Omega(1633+\Delta)$, $d^{-1}_\Omega(1813+\Delta)$,
3. Charmed Quarks [The Ground Quark is $\mathbf{u^1_C(1753+\Delta)}$]
Δ-Axis: $\mathbf{u^1_C(1753+\Delta)}$, $u^1_{\rm C}(6073+\Delta)$, $u^1_C(13273+\Delta)$.
D-Axis: $u^1_{\rm C}(2133+\Delta)$,$u^1_{\rm C}(2333+\Delta)$,$u^1_{\rm C}(2533+\Delta)$,$u^1_{\rm C}(3543+\Delta)$.
F-Axis: $q^1_{\Xi_C}(1873+\Delta)$, $q^1_{\Xi_C}(2163+\Delta)$, $q^1_{\Xi_C}(2363+\Delta)$, $q^1_{\Xi_C}(3193+\Delta)$.
G-Axis: $d^1_{\Omega_C}(2133+\Delta)$, $d^1_{\Omega_C}(2513+\Delta)$, $d^1_{\Omega_C}(3253+\Delta)$.
4. Bottom Quarks [The Ground Quark is $\mathbf{d^1_b(4913+\Delta)}$]
Σ-Axis: $\mathbf{d^1_b(4913+\Delta)}$, $d^1_b(9333+\Delta)$, $d^1_b(15193+\Delta)$, $d^1_b(22493+\Delta)$, ...

In fact, there are three completely identical tables (as Table 11) with the red, the yellow and the blue colors respectively. Omitting the color, we use one of the three tables to represent all three tables.

For the four flavored quarks, there are five ground quarks [u(313+Δ), d(313+Δ), d_S(493+Δ), u_C(1753+Δ) and d_b(4913+Δ)] in the quark spectrum. They correspond with the five quarks [10] of the current Quark Model: u $\leftrightarrow$ u(313+Δ), d $\leftrightarrow$ d(313+Δ), s $\leftrightarrow$ d_S(493+Δ), c $\leftrightarrow$ u_C(1753+Δ) and b $\leftrightarrow$ d_b(4913+Δ). They are all the energy band excited states of the elementary quarks ϵ. The quarks u(313+Δ) and u_C(1753+Δ) (with Q = $+\frac{2}{3}$) are the excited states of the elementary ϵ_u-quark. The quarks d(313+Δ), d_S(493+Δ) and d_b(4913+Δ) (with Q = -$\frac{1}{3}$) are the excited states of the elementary ϵ_d-quark.

There are the values of the current quarks that can compare with the deduced values of the five ground quarks. The deduced intrinsic quantum numbers (I, S, C, b and Q) of the five ground quarks are exactly the same as the five current quarks (see Table 10 and [10]). Table 11* shows that

Table 11* The Comparison of Deduce Ground Quarks and Current Quarks

	u(m (Mev))	d(m (Mev))	u(m (Mev))	u(m (Mev))	u(m (Mev))
Current	u(1.5 to 4)	d(4 to 8)	s(80 to 130)	c(1250 to 1350)	b(4100 to 4900)
Deduced	u(313)	d(313)	d_s(493)	u_c(1753)	d_b(4913)
Δm	310	307	388	493	487

the deduced rest masses of the five ground quarks are roughly a constant (about 400 Mev) larger than the masses of the current quarks. These can be originated from different energy reference systems. If we use the same energy reference system, the deduced masses of ground quarks are roughly consistent with the masses of the corresponding current quarks. Of course the ultimate test is whether or not the baryons and mesons that are composed by the quarks are consistent with the experimental results.

Now we have deduced a quark (excited from the vacuum) spectrum. We have found

the intrinsic quantum numbers (S, C, b and Q) and rest masses of the quarks. From these rest masses and the quantum numbers, we can deduce a baryon spectrum and a meson spectrum using sum laws and phenomenological binding energy formulae,

IV The Baryon Spectrum

According to the Quark Model [1], a baryon is composed of three quarks with different colors. For each flavor, the three different colored quarks have the same I, S, C, b, Q and rest mass. Thus, we can omit the color when we deduce the rest masses and intrinsic quantum numbers of the baryons. We must remember, however, that three colored quarks (red, yellow and blue) compose a colorless baryon. Since we have already found the quarks, we can use sum laws to find the intrinsic quantum numbers (I, S, C, b and Q) of baryons (qqq) and the rest masses of all baryons, except those of the charmed baryons. We can deduce the rest masses of the charmed baryons with a phenomenological formula (55). There are more than 80 quarks in Table 11. They can make many possible baryons, much more than the experimental baryons [11]. Since the probabilities that three quarks make a baryon are different, the experimentally discovered baryons are the possible baryons with larger observable probabilities. The observable probability depends on the following factors: 1) the ground quarks have higher probabilities of occurrence than other quarks have, 2) the lower rest mass quarks have higher probabilities of occurrence, 3) the quarks with lower isospin have more possibilities of occurrence than the higher isospin quarks, 4) the quarks born on the symmetry axes with more symmetry operations have higher probabilities of occurrence. If we use P(q_{name}(m)) to represent the probability of a q_{name}(m) forming a baryon, we assume:

$$\begin{array}{l} P(u(313)) \sim P(d(313)) >> \\ P(d_S(493)) > P(u_C(1753)) > P(d_b(4913)) >> \\ P(u_C(m)) > P(d_b(m)) > P(d_S(m)) > P(q_N(m)) > P(d_\Sigma(m)) > \\ P(q_\Delta(m)) > P(d_\Omega(m)) > P(d_{\Xi_C}(m)) > P(d_{\Omega_C}(m)) > P(d_\Xi(m)) >. \end{array} \tag{47}$$

From (47) we find that the possible baryons (with none or only one ground q(313)) have much lower observable probabilities than baryons with two ground quarks (or three ground q(313) quarks). Thus, we omit the baryons that have only one ground quark q(313) or do not have any ground quark q(313). We will show that the possible baryons with two ground quarks [q_1(m)q(313)(q(313)] can explain the experimental baryon spectrum.

A The Intrinsic Quantum Numbers of the Baryons

A1. The intrinsic quantum numbers of a baryon are the sums of the three constituent quarks (q_1(m) + q_2(313) + q_3(313)). We can find the strange number S_B, the charmed number C_B, the bottom number b_B and the electric charge Q_B of the baryons by

$$\begin{aligned}
&S_B = S_{q_1} + S_{q_{N(313)}} + S_{q_{N(313)}} = S_{q_{1(m)}},\\
&C_B = C_{q_1} + C_{q_{N(313)}} + C_{q_{N(313)}} = C_{q_1(m)},\\
&b_B = b_{q_1} + b_{q_{N(313)}} + b_{q_{N(313)}} = b_{q_{1(m)}},\\
&Q_B = Q_{q_1} + Q_{q_{N(313)}} + Q_{q_{N(313)}}.
\end{aligned} \tag{48}$$

Since the ground quark q(313) has S = C = b = 0 , the baryon [q_1(m)+q_2(313)+q_3(313)] has the same S, C and b as the q_1(m)-quark has.

A2. The Isospin I_B of the baryon ($q_1q_2q_3$) is found by

$$\overrightarrow{I_B} = \overrightarrow{I_{q_1(m)}} + \overrightarrow{I_{q_2}} + \overrightarrow{I_{q_3}}. \tag{49}$$

Since $I_{q_2} = I_{q_3} = I_{q(313)} = \frac{1}{2}$ and the top limit of the experimental isospin values of baryons is $\frac{3}{2}$, the isospins of the baryons (q_1q_2 q_3) are:

$$\begin{aligned}
&\text{for } I_{q_1} = \tfrac{3}{2} \rightarrow I_B = \tfrac{1}{2}, \tfrac{3}{2};\\
&\text{for } I_{q_1} = 1 \rightarrow I_B = 0, 1;\\
&\text{for } I_{q_1} = \tfrac{1}{2} \rightarrow I_B = \tfrac{1}{2}, \tfrac{3}{2};\\
&\text{for } I_{q_1} = 0 \rightarrow I_B = 0, 1.
\end{aligned} \tag{50}$$

After having the baryon isospin I_B, we can get the $I_{B,\,z}$ using the follow formula

$$\begin{array}{ll} \text{for } I_B = \frac{3}{2}, & I_{B,\,z} = \frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}; \\ \text{for } I_B = 1, & I_{B,\,z} = 1, 0, -1; \\ \text{for } I_B = \frac{1}{2}, & I_{B,\,z} = \frac{1}{2}, -\frac{1}{2}; \\ \text{for } I_B = 0, & I_{B,\,z} = 0. \end{array} \quad . \tag{51}$$

Using R values (see Table A3) and equivalent $\vec{n}$ values (66), we can find the maximum isospin value of quarks for each symmetry point. The probability that a quark (q_1) with lower isospin forms a baryon [$q_1(m)q_2(313)q_3(313)$] with higher isospin is low. Thus, there is very low possibility that a baryon has a higher isospin than the maximum I of quarks have on the point. Since all symmetry axes at the point N are two fold, the maximum I of the baryon is $\frac{1}{2}$ at the N-point. We show the maximum I values of the quarks and baryons in Table A6.

A3. For the baryon [$q_1(m)q_2(313)q_3(313)$], we can deduce the isospin I_B and $I_{B,Z}$ using (50) and (51). For the $q_1(m)$-quark, I_Z can be found in Table 10. The quark $q_1(m)$ selects two ground quarks q_2 and q_3 from u(313)u(313), u(313)d(313) and d(313)d(313), using the follow formulae:

$$I_{Z,B} = I_{Z,q_1(m)} + I_{Z,q_2} + I_{Z,q_3}, \tag{52}$$

$$Q_B = Q_{q_1} + Q_{q_2} + Q_{q_3}, \tag{53}$$

to get the correct $I_{Z,B}$ and Q_B. We show selected results in Table 12:

Table 12. The Quark Constitutions of the Baryons

$B^{I_z,\,B}_{aryon}$	$N^{\frac{1}{2}}$	$N_N^{\frac{-1}{2}}$	$\Delta_\Delta^{\frac{3}{2}}$	$\Delta_\Delta^{\frac{1}{2}}$	$\Delta_\Delta^{\frac{-1}{2}}$	$\Delta_\Delta^{\frac{-3}{2}}$	Σ_Σ^{1}	Σ_Σ^{0}	Σ_Σ^{-1}
I_B	1/2	1/2	3/2	3/2	3/2	3/2	1	1	1
$q_1^{I_Z}$ (u^{I_Z} or d^{I_z})	$u_N^{\frac{1}{2}}$	$d_N^{\frac{-1}{2}}$	$u_\Delta^{\frac{1}{2}}$	$u_\Delta^{\frac{1}{2}}$	$d_\Delta^{\frac{-1}{2}}$	$d_\Delta^{\frac{-1}{2}}$	u_Σ^{1}	d_Σ^{0}	d_Σ^{0}
$q_2^{I_z}$	$u^{\frac{1}{2}}$	$u^{\frac{1}{2}}$	$u^{\frac{1}{2}}$	$u^{\frac{1}{2}}$	$u^{\frac{1}{2}}$	$d^{\frac{-1}{2}}$	$u^{\frac{1}{2}}$	$u^{\frac{1}{2}}$	$d^{\frac{-1}{2}}$
$q_3^{I_z}$	$d^{\frac{-1}{2}}$	$d^{\frac{-1}{2}}$	$u^{\frac{1}{2}}$	$d^{\frac{-1}{2}}$	$d^{\frac{-1}{2}}$	$d^{\frac{-1}{2}}$	$d^{\frac{-1}{2}}$	$d^{\frac{-1}{2}}$	$d^{\frac{-1}{2}}$
$I_{Z,B}=\Sigma I_{z,q_i}$	$\frac{1}{2}$	$\frac{-1}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{-1}{2}$	$\frac{-3}{2}$	1	0	-1
$Q_B=\Sigma Q_{q_i}$	1	0	2	1	0	-1	1	0	-1
$S_B= S_{q_1}$	0	0	0	0	0	0	-1	-1	-1
$C_B= C_{q_1}$	0	0	0	0	0	0	0	0	0
$b_B= b_{q_1}$	0	0	0	0	0	0	0	0	0
$B^{Q_B}_{aryon}$	N^+	N^0	Δ^{++}	Δ^+	Δ^0	Δ^-	Σ^+	Σ^0	Σ^-
*********	***	***	***	***	***	***	***	***	***
$B^{I_z,B}_{aryon}$	$\Xi_\Xi^{\frac{1}{2}}$	$\Xi_\Xi^{\frac{-1}{2}}$	Λ_S^0	Λ_b^0	Ω^0	$\Xi_C^{\frac{1}{2}}$	$\Xi_C^{\frac{-1}{2}}$	Λ_C^0	Ω_C^0
I_B	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0	0
$q_{q_1}^{I_Z}$ (u^{I_Z},d^{I_z})	$u_\Xi^{\frac{3}{2}}$	$d_\Xi^{\frac{1}{2}}$	d_S^0	d_b^0	d_Ω^1	$u_{\Xi_C}^{\frac{1}{2}}$	$d_{\Xi_C}^{\frac{-1}{2}}$	u_C^0	$d_{\Omega_C}^0$
$q_2^{I_z}$	$d^{\frac{-1}{2}}$	$d^{\frac{-1}{2}}$	$u^{\frac{1}{2}}$	$u^{\frac{1}{2}}$	$d^{\frac{-1}{2}}$	$u^{\frac{1}{2}}$	$u^{\frac{1}{2}}$	$u^{\frac{1}{2}}$	$u^{\frac{1}{2}}$
$q_3^{I_z}$	$d^{\frac{-1}{2}}$	$d^{\frac{-1}{2}}$	$d^{\frac{-1}{2}}$	$d^{\frac{-1}{2}}$	$d^{\frac{-1}{2}}$	$d^{\frac{-1}{2}}$	$d^{\frac{-1}{2}}$	$d^{\frac{-1}{2}}$	$d^{\frac{-1}{2}}$
$I_{Z,B}=\Sigma I_{z,q_i}$	$\frac{1}{2}$	$\frac{-1}{2}$	0	0	0	$\frac{1}{2}$	$\frac{-1}{2}$	0	0
$Q_B=\Sigma Q_{q_i}$	0	-1	0	0	-1	1	0	1	0
$S_B= S_{q_1}$	-2	-2	-1	0	-3	-1	-1	0	-2
$C_B= C_{q_1}$	0	0	0	0	0	1	1	1	1
$b_B= b_{q_1}$	0	0	0	-1	0	0	0	0	0
$B^{Q_B}_{aryon}$	Ξ^0	Ξ^-	Λ_S^0	Λ_b^0	Ω^{-1}	Ξ_c^+	Ξ_c^0	Λ_C^+	Ω_C^0

Table 12 also shows the intrinsic quantum numbers (I, S, C, b and Q) of the baryons. The baryons that we deduced from the phenomenological formulae have exactly the same intrinsic quantum numbers as the experimental results.

B The Binding Energy of the Three Quarks Inside a Charmed Baryon

Since all quarks inside baryons are the excited states of the same elementary quarks ϵ, the binding energies of the quarks inside the baryons are the strong interaction energy of the three colors. Because the three colors are exaetly the same for all baryons, the baryon binding energies are the same for all baryons. The baryons binding energy of all baryons is a roughly unknown constant (-3Δ, $\Delta = \frac{1}{3}|E_{B-B}|$ (34)). Thus, the rest mass (M) of a baryon will be

$$M = m^*_{q_1} + m^*_{q_2} + m^*_{q_3} - 3\Delta.$$

Because a quark has mass m^*_q (from Table 11),

$$m^*_q = m_q + \Delta,$$

we have a baryon mass,

$$\begin{aligned} M_B &= m^*_{q_1} + m^*_{q_2} + m^*_{q_3} - 3\Delta \\ &= m_{q_1} + m_{q_2} + m_{q_3}, \end{aligned} \tag{54}$$

where $m_q = m^*_q - \Delta$. The above formula (54) means that the rest mass of a baryon equals the sum of the three quark rest masses (m). Since the binding energy (-3Δ) is always cancelled by the three (Δ) of the three quark masses, we can omit the Δ in the rest mass of the three quarks inside the baryon and omit the (-3Δ) in the binding energy of the baryon when we count the rest masses of the baryons. In fact, the interaction energies of the colors are very complex; (-3Δ) is only a phenomenological approximation of the binding energy.

The mass sum law (54) is valid for most baryons except the charmed baryons. For charmed baryons, however, we shall add a small amount of adjustment in energy, Δe, to the large unknown constant (3Δ):

$$\Delta e = 100C\ [2I - 1 - \frac{1}{2}\Theta(1+S_{Ax})] \ \ (\text{Mev}), \tag{55}$$

where C is the charmed number of the baryon, I is the isospin of the baryon and S_{Ax} is the strange number of the axis of the charmed quark (q_1). The in-out number Θ is defined in (65). From Table A3, $\Theta = 0$ for the Δ-axis, the Λ-axis and the Σ-axis, while $\Theta = 1$ for the D-axis, the F-axis and the G-axes. From (54) and (55), for charmed baryons,

$$M_B = m_{q_1} + m_{q_2} + m_{q_3} + \Delta e \tag{56}$$

From the quark spectrum (Table 11), using the sum laws (48), (54) and (49) [and the binding energy formula (56) for the charmed baryons only], we can deduce the baryons.

C Deduction of the Baryons

We have already found the intrinsic quantum numbers of the baryons that are shown in Table 12. If we can deduce the rest masses of baryons, we will deduce the baryon spectrum. We give three types of examples to show how to deduce the baryons.

C1. The Deduction of Charmed Baryons on the Δ-axis (Γ-H)

In this example, we will deduce the charmed baryons on the Δ-axis, the Σ-axis and the Λ-axis. In fact, all charmed baryons are only on the Δ-axis. There is not any charmed baryons on the other two axes.

For the Δ-axis, $S_\Delta = 0$ and $\Theta = 0$ from Table 3; from (55),

$$\Delta e = 100 \times C(2I\text{-}1) \quad (\text{Mev}) \tag{57}$$

From Table 4, we have d_S(m) and u_C(m). From (56) and (57), we have:

$$\text{for the quark } u_C(m) \rightarrow \left\{ \begin{array}{l} u_C(m)+u(313)+d(313)] = \Lambda_C(m+626+\Delta e) \text{ and} \\ u_C(m)+q_N(313)+q_N(313)] = \Sigma_c(m+626+\Delta e) \end{array} \right\};$$

$$\text{for the quark } d_S(m) \rightarrow \left\{ \begin{array}{l} d_S(m)+u(313)+ d(313) = \Lambda(m+626) \text{ and} \\ d_S(m)+q_N(313)+ q_N(313) = \Sigma(m+626) \end{array} \right\}.$$

Table 13 shows the Λ_c-baryons, the Σ_c-baryons, the Λ-baryons and the Σ-baryons:

Table 13. The Λ_c-Baryons and the Σ_c-Baryons

E_{Point}	q_{name}(m)	$m_{q_2+q_3}$	Δe_Λ	Λ(M(Mev))	Δe	Σ(M(Mev))
E_H=1	d_S(773)	626	0	Λ(1399)	0	Σ(1399)
E_Γ=4	u_C(1753)	626	-100	Λ_c(2279)	100	Σ_c(2479)
E_H=9	d_S(3753)	626	0	Λ(4379)	0	Σ(4379)
E_Γ=16	u_C(6073)	626	-100	Λ_c(6599)	100	Σ_c(6799)
E_H=25	d_S(9613)	626	0	Λ(10239)	0	Σ(10239)
E_Γ=36	u_C(13273)	626	-100	Λ_c(13799)	100	Σ_c(13999)

C2. Deduction of the Charmed Baryons on the D-axis, the F-axis and the G-axis

In this example, we will deduce low-rest-mass charmed baryons on the D-axis, F-axis and G-axis. From Table B5 (the D-axis), we obtain the charmed quarks u_C(2133), u_C(2333), u_C(2533) and u_C(3543). From Table B6 (the F-axis), we obtain the charmed strange quarks q_{Ξ_C}(1873), q_{Ξ_C}(2163), q_{Ξ_C}(2363) and q_{Ξ_C}(3163). From Table B7 (the G-axis), we obtain the charmed strange quarks q_{Ω_C}(2133) and q_{Ω_C}(2513). Using the sum laws (48) and the binding energies of the charmed baryons (56), we have:

Table 14. The Charmed Baryons on the D, F and G Axis

Axes	S_{AX}	Θ	C	I	$Quark_1$	$m_{q_2}+m_{q_3}$	Δe	Baryon
D	0	1	1	0	u_C(2133) u_C(2333) u_C(2533) u_C(3543)	626	-150	Λ_C(2609) Λ_C(2809) Λ_C(3009) Λ_C(4019)
F	-1	1	1	$\frac{1}{2}$	q_{Ξ_C}(1873) q_{Ξ_C}(2163) q_{Ξ_C}(2363) q_{Ξ_C}(3193)	626	0	Ξ_C(2499) Ξ_C(2789) Ξ_C(2989) Ξ_C(3819)
G	-2	1	1	0	q_{Ω_C}(2133) q_{Ω_C}(2513)	626	-50	Ω_C(2709) Ω_C(3089)

C3. Deduction of the Uncharmed Baryons (C = 0)

For uncharmed baryons, Table 12 has already given the intrinsic quantum numbers. Since C = 0→ Δe =0, from (54), the rest masses of the baryons $M_B = m_{q_1} + m_{q_2} + m_{q_3}$.

Example 1. We deduce the baryons on the fourfold energy bands of the Δ-axis. For the fourfold energy bands, from Table B1, C = 0; thus, Δe = 0 from (55). With (50), we can get two kinds of baryons (the N-baryons with I = $\frac{1}{2}$ and the Δ-baryons with I = $\frac{3}{2}$). From Table 3, we have q_Δ(m) (q_1 of $q_1q_2q_3$). Using Table 12, we get q_2 and q_3. From (54), we get the baryon mass $M = m_{q_\Delta} + m_{q_2} + m_{q_3} = m_{q_\Delta}$+626(Mev) → $\Delta(M{=}m_{q_\Delta}+626)$ and $N(M{=}m_{q_\Delta}+626)$:

Table 15 A. The Δ-Baryons and the N-Baryons on the Δ-axis

E_{Point}	$E(\vec{k},\vec{n})$	I_{q_1}	q_{Name}(m)	$m_{q_2+q_3}$	I_Δ	Δ(M)	I_N	N(M)
E_H=1	673	$\frac{3}{2}$	q_Δ(673)	626	$\frac{3}{2}$	Δ(1299)	$\frac{1}{2}$	N(1299)
E_Γ=2	1033	$\frac{3}{2}$	q_Δ(1033)	626	$\frac{3}{2}$	Δ(1659)	$\frac{1}{2}$	N(1659)
	1033	$\frac{3}{2}$	q_Δ(1033)	626	$\frac{3}{2}$	Δ(1659)	$\frac{1}{2}$	N(1659)
E_H=3	1393	$\frac{3}{2}$	q_Δ(1393)	626	$\frac{3}{2}$	Δ(2019)	$\frac{1}{2}$	N(2019)
E_Γ=4	1753	$\frac{3}{2}$	q_Δ(1753)	626	$\frac{3}{2}$	Δ(2379)	$\frac{1}{2}$	N(2379)
E_H=5	2113	$\frac{3}{2}$	q_Δ(2113)	626	$\frac{3}{2}$	Δ(2739)	$\frac{1}{2}$	N(2739)
	2113	$\frac{3}{2}$	q_Δ(2113)	626	$\frac{3}{2}$	Δ(2739)	$\frac{1}{2}$	N(2739)
E_H=5	2113	$\frac{3}{2}$	q_Δ(2113)	626	$\frac{3}{2}$	Δ(2739)	$\frac{1}{2}$	N(2739)
E_H=5	2113	$\frac{3}{2}$	q_Δ(2113)	626	$\frac{3}{2}$	Δ(2739)	$\frac{1}{2}$	N(2739)
E_Γ=6	2473	$\frac{3}{2}$	q_Δ(2473)	626	$\frac{3}{2}$	Δ(3099)	$\frac{1}{2}$	N(3099)
	2473	$\frac{3}{2}$	q_Δ(2473+Δ)	626	$\frac{3}{2}$	Δ(3099)	$\frac{1}{2}$	N(3099)
E_Γ=6	2473	$\frac{3}{2}$	q_Δ(2473+Δ)	626	$\frac{3}{2}$	Δ(3099)	$\frac{1}{2}$	N(3099)

Example 2. We deduce the N(M)-baryons and the Δ(M)-baryons on the D-axis. From Table B5, we get the q_N(m)-quarks. As with example 1, q_N(m)→ $N(M{=}m_{q_\Delta}+626)$ and $\Delta(M{=}m_{q_\Delta}+626)$ at point p; from Table A6, q_N(m)→$N(M{=}m_{q_\Delta}+626)$ at point N:

Table 15B. The N(m)-baryons and the Δ(M)-baryons on the D-axis

E_{Start}	E	q_N(m)	$m_{q_2+q_3}$	I_B	N(M)	I_B	Δ(M)
$E_P = \frac{3}{4}$	583	$q_N^0(583)$	626	$\frac{1}{2}$	N(1209)	$\frac{3}{2}$	Δ(1209)
$E_N = \frac{3}{2}$	853	$q_N^0(853)$	626	$\frac{1}{2}$	N(1479)		
$E_N = \frac{5}{2}$	1213	$2q_N^0(1213)$	626	$\frac{1}{2}$	N(1839)		
$E_P = \frac{11}{4}$	1303	$2q_N^0(1303)$	626	$\frac{1}{2}$	N(1929)	$\frac{3}{2}$	2Δ(1929)
$E_N = \frac{7}{2}$	1573	$2q_N^0(1573)$	626	$\frac{1}{2}$	N(2199)		
$E_p = \frac{19}{4}$	2023	$q_N^0(2023)$	626	$\frac{1}{2}$	N(2649)	$\frac{3}{2}$	Δ(2649)
$E_P = \frac{19}{4}$	2023	$2q_N^0(2023)$	626	$\frac{1}{2}$	N(2649)	$\frac{3}{2}$	2Δ(2649)
$E_N = \frac{11}{2}$	2293	$q_N^0(2293)$	626	$\frac{1}{2}$	N(2919)		
$E_N = \frac{13}{2}$	2653	$2q_N^0(2653)$	626	$\frac{1}{2}$	N(3279)		

We have already deduced low mass charmed baryons in Table 13 and Table 14. Using sum laws, we deduced some uncharmed baryons in Table 15A and Table 15b. Continuing the above procedure, we can deduce a baryon spectrum that is shown in Table 16-Table 21.

D Comparing with the Experimental Results of Baryons

Using Table 16 – Table 21, we can compare the deduced baryon spectrum with the experimental results [11]. In this comparison, we do not take into account the angular momenta of the experimental results. We assume that the small differences of the masses in the same group of baryons with the same quantum numbers (I, S, C, b and Q) are from their different angular momenta. If we ignore this effect, their masses would be essentially the same. Since all baryons in the group have the same intrinsic quantum numbers with the same name, we use the baryon name to represent the intrinsic quantum numbers, as shown in the second column of Table 16. If the name is the same between the deduced baryon and the experimental baryon, this means that the intrinsic quantum numbers (I, S, C, b and Q) are exactly the same. We use the baryons with the average rest mass of the group of baryons (see Table C1) to represent the group of the baryons.

The mass units of quarks and baryons as well as the widths are “Mev” :

Table 16. The Ground Baryons.

Deduced	Quantum No.	Experiment	$\frac{\Delta M}{M}$%	Lifetime
Name(M)	S, C, b, I, Q	Name(M)		
p(939)	0, 0, 0, $\frac{1}{2}$, 1	p(938)	0.11	>10^{29} *years*
n(939)	0, 0, 0, $\frac{1}{2}$, 0	n(940)	0.11	885.7 s
Λ^0(1119)	-1, 0, 0, 0, 0	Λ^0(1116)	0.27	2.6× 10^{-10}s
Σ^+(1209)	-1, 0, 0, 1, 1	Σ^+(1189)	1.7	.80× 10^{-10}s
Σ^0(1209)	-1, 0, 0, 1, 0	Σ^0(1193)	1.4	7.4× 10^{-20}s
Σ^-(1209)	-1, 0, 0, 1, -1	Σ^-(1197)	1.0	1.5×10^{-10}s
Ξ^0(1299)	-2, 0, 0, $\frac{1}{2}$, 0	Ξ^0(1315)	1.2	2.9×10^{-10}s
Ξ^-(1299)	-2, 0, 0, $\frac{1}{2}$, -1	Ξ^-(1321)	1.7	1.6×10^{-10}s
Ω^-(1659)	-3, 0, 0. 0, -1	Ω^-(1672)	0.78	.82×10^{-10}s
Λ_c^+(2279)	0, 1, 0, 0, 1	Λ_c^+(2285)	0.26	200×10^{-15}s
Ξ_c^+(2499)	-1, 1, 0, $\frac{1}{2}$, 1	Ξ_c^+(2466)	1.4	442×10^{-15}
Ξ_c^0(2499)	-1, 1, 0, $\frac{1}{2}$, 0	Ξ_c^0(2472)	1.2.	112×10^{-15}s
Ω_C^0(2709)	0, 0, -1, 0, 0	Ω_c^0(2698)	0.41	69×$10^{-15}s$
Λ_b^0(5539)	0, 0, -1, 0, 0	Λ_b^0(5624)	1.5	1.23$10^{-12}s$
Σ_C^{++}(2479)	-1, 1, 0, 1, 2	Σ_C^{++}(2453)	1.1	Γ=2.2 Mev
Σ_C^+(2479)	-1, 1, 0, 1, 1	Σ_C^{++}(2451)	1.2	Γ<4.6 Mev
Σ_C^0(2479)	-1, 1, 0, 1, 0	Σ_C^{++}(2452)	1.2	Γ=2.1 Mev

The most important baryons are shown in Table 16. These baryons have relatively long lifetimes. They are the most important experimental results of the baryons. Their deduced intrinsic quantum numbers are the same as the experimental results. The deduced mass values are over 98% consistent with the experimental values.

Two kinds of the strange baryons Λ and Σ are compared in Table 17. Their deduced intrinsic quantum numbers are the same as the experimental results. The deduced masses of the baryons Λ and Σ are about 98% consistent with the experimental results.

Table 17. Two Kinds of Strange Baryons Λ and Σ ($S = -1$)

Deduced	Experiment, Γ	$\frac{\Delta M}{M}$%	Deduced	Experiment, Γ	$\frac{\Delta M}{M}$%
$\Lambda(1119)$	$\Lambda(1116)$	0.36	$\Sigma(1209)$	$\Sigma(1193)$	1.4
$\Lambda(1399)$	$\Lambda(1405)$,50	0.43	$\Sigma(1399)$	$\Sigma(1385)$,37	1.0
$\Lambda(\overline{1659})$	$\Lambda(\overline{1620}),\overline{65}$	2.4	$\Sigma(\overline{1727})$	$\Sigma(\overline{1714}),\overline{93}$	0.76
$\Lambda(\overline{1889})$	$\Lambda(\overline{1830}),\overline{145}$	3.2	$\Sigma(\overline{1929})$	$\Sigma(\overline{1928}),\overline{170}$	0.05
$\Lambda(\overline{2079})$	$\Lambda(\overline{2105}),\overline{200}$	1.24	$\Sigma(\overline{2019})$	$\Sigma(\overline{2045}),\overline{220}$	1.3
$\Lambda(\overline{2339})$	$\boldsymbol{\Lambda}(\overline{2338})$**,** $\overline{159}$	0.04	$\Sigma(\overline{2249})$	$\Sigma(2250)$,100	0.05
$\Lambda(\overline{2615})$	$\Lambda(2585)^*$,300	1.2	$\Sigma(\overline{2492})$	$\Sigma(2455)^*$,140	1.1
$\Lambda(3099)$	Prediction		$\Sigma(\overline{2644})$	$\Sigma(2620)^*$,200	0.84
$\Lambda(3369)$	Prediction		$\Sigma(3099)$	$\Sigma(3085)^*$, ?	0.45
			$\Sigma(3369)$	Prediction	

*Evidences of existence for these baryons are only fair; they are not listed in the Baryon Summary Table [11].

Table 18 compares the deduced results with the experimental results of the unflavored baryons N and Δ:

Table 18. The Unflavored Baryons N and Δ (S= C=b = 0)

Deduced	Experiment, Γ	$\frac{\Delta M}{M}$%	Deduced	Experiment, Γ	$\frac{\Delta M}{M}$%
N(939)	N(939)	0.0			
N($\overline{1254}$)	Covered by $\Delta(1232)$		$\Delta(1254)$	$\Delta(1232)$,120	1.8
N(1479)	N($\overline{1498}$),$\overline{207}$	1.3			
N($\overline{1650}$)	N($\overline{1689}$),$\overline{130}$	2.3	$\Delta(\overline{1659})$	$\Delta(\overline{1640}),\overline{267}$	1.2
N($\overline{1929}$)	N($\overline{1912}$),440	0.31	$\Delta(\overline{1955})$	$\Delta(\overline{1923}),\overline{264}$	1.7
N($\overline{2199}$)	N($\overline{2220}$),$\overline{417}$	0.95			
N(2379)	Covered by $\Delta(2420)$		$\Delta(2379)$	$\Delta(2420)$,400	1.7
N($\overline{2649}$)	N(2600),650	1.9			
N(2739)	N(2700)*,600	1.5	$\Delta(\overline{2694})$	$\Delta(2750)^*$,400	2.0
N(2919)	Prediction		$\Delta(3099)$	$\Delta(\overline{2975})^*,\overline{750}$	
N(3099)	Prediction		$\Delta(3369)$	Prediction	

*Evidences are fair, they are not listed in the Baryon Summary Table [11].

The deduced masses of the baryons N and Δ are 98% consistent with the experimental results. We do not find the deduced N(1209) and N(1299) in the experiment results. We believe that they are covered up by the experimental baryon $\Delta(1232)$ because of the following reasons: (1) they are unflavored baryons with the same S, C and b; (2) the width (120 Mev) of $\Delta(1232)$ is very large, and the baryons $N(1209)$ and $N(1299)$ both fall within the width region of $\Delta(1232)$; (3) the average mass (1255 Mev) of $N(1209)$ and $N(1299)$ is essentially the same as the mass (1232 Mev) of $\Delta(1232)$ ($\Gamma = 120$ Mev).

The deduced intrinsic quantum numbers of the baryons Ξ and Ω are the same as the experimental results (see Table 19). The deduced masses of the baryons Ξ and Ω are more than 98% compatible with experimental results.

Table 19. The Baryons Ξ and the Baryons Ω

Deduced	Experiment, Γ	$\frac{\Delta M}{M}$%	Deduced	Experiment, Γ	$\frac{\Delta M}{M}$%
$\Xi(1299)$	$\Xi(1318)$	1.4	$\Omega(1659)$	$\Omega(1672)$	**0.7**
$\Xi(1479)$	$\Xi(1530)$, 9.9	3.3	$\Omega(2259)$	$\Omega(2252)$, 55	0.4
$\Xi(1659)$	$\Xi(1690)$, <30	1.8	$\Omega(2439)$	$\Omega(\overline{2427})^*$, $\overline{48}$	0.5
$\Xi(1839)$	$\Xi(1823)$, 24	1.1	$\Omega(2979)$	Prediction	
$\Xi(2019)$	$\Xi(\overline{1988})^{\#}$,$\overline{40}$	1.6	$\Omega(3819)$	Prediction	
$\Xi(2199)$	$\Xi(\overline{2185})^{\$}$,33	0.64			
$\Xi(\overline{2369})$	$\Xi(2370)^*$,80	0.04	$\Omega(\overline{2427})^*$	$=\frac{1}{2}[\Omega(2380)^*+\Omega(2474)^*]$	
$\Xi(\overline{2549})$	$\Xi(2500)^*$,150	2.0	$\Xi(\overline{1988})^{\#}$	$=\frac{1}{2}[\Xi(1950)+\Xi(2025)]$	
$\Xi(2759)$	Prediction		$\Xi(\overline{2185})^{\$}$	$=\frac{1}{2}[\Xi(\overline{2120})^*+\Xi(\overline{2250})^*]$	

*Evidences of existence for these baryons are only fair: they are not listed in the Baryon Summary Table [11].

We compare the charmed Λ_c^+-baryon, $\mathbf{\Omega}_C$-baryon and Λ_b^0-baryon in Table 20. Their deduced intrinsic quantum numbers are the same as the experimental results. The experimental masses of the charmed baryons (Λ_c^+ and $\mathbf{\Omega}_C$) and bottom baryons (Λ_b^0) coincide more than 98% with the deduced results.

Table 20. The Λ_c^+-Baryons, the Λ_b^0-Baryons and the $\boldsymbol{\Omega}_C$-Baryons

Deduced	Experiment	$\frac{\Delta \mathrm{M}}{\mathrm{M}}\%$	Deduced	Experiment	$\frac{\Delta \mathrm{M}}{\mathrm{M}}\%$
$\Lambda_c^+(\mathbf{2279})$	$\Lambda_c^+(\mathbf{2285})$	0.2	$\Lambda_b^0(5539)$	$\Lambda_b^0(5624)$	1.5
$\Lambda_C^+(\mathbf{2609})$	$\Lambda_C^+(2594)$ $\Lambda_C^+(2627)$ $\Lambda_c^+(\overline{\mathbf{2611}})$	0.46	$\Lambda_b^0(9959)$	Prediction	
$\Lambda_C^+(\mathbf{2809})$	$\Lambda_C^+(2765)^*$ $\Lambda_C^+(2881)^*$ $\Lambda_c^+(\overline{\mathbf{2823}})^*$	0.85.			
$\Lambda_C^+(\mathbf{3009})$	Prediction		$\boldsymbol{\Omega}_C(\mathbf{2709})$	$\boldsymbol{\Omega}_C(\mathbf{2698})$	0.44
$\Lambda_C^+(\mathbf{4019})$	Prediction		$\Omega_C(\mathbf{3089})$	Prediction	

*Evidences of existence for these baryons are only fair;
they are not list in the Baryon Summary Table [11].

Finally we compare the deduced results with the experimental results for the charmed strange baryons Ξ_c and Σ_c in Table 21. Their intrinsic quantum numbers all match exactly and their masses also agree well (more than 98%).

Table 21. Charmed Strange Baryon Ξ_c and Σ_c

Deduced	Experiment	$\frac{\Delta \mathrm{M}}{\mathrm{M}}\%$	Deduced	Experiment	$\frac{\Delta \mathrm{M}}{\mathrm{M}}\%$
$\Xi_c(\mathbf{2499})$	$\Xi_C(2469)$ $\Xi_C(2576)$ $\Xi_c(\overline{\mathbf{2523}})$	1.0.	$\boldsymbol{\Sigma}_c(\mathbf{2479})$	$\Sigma_c(2452)$ $\Sigma_c(2518)$ $\boldsymbol{\Sigma}_c(\overline{\mathbf{2485}})$	0.24
$\Xi_C(\mathbf{2789})$	$\Xi_C(2645)$ $\Xi_C(2790)$ $\Xi_C(2815)$ $\Xi_c(\overline{\mathbf{2750}})$	**1.4**	$\boldsymbol{\Sigma}_c(\mathbf{6799})$	Prediction	
$\Xi_C(\mathbf{2989})$	Prediction				
$\Xi_C(\mathbf{3819})$	Prediction				

*Evidences of existence for these baryons are only fair; they are not listed in the Baryon Summary Table [11].

In summary, the phenomenological formulae explain all baryon experimental intrin-

sic quantum numbers (100%) and the rest masses (about 98%). We explain virtually all experimentally-confirmed baryons in this paper.

V The Meson Spectrum

According to the Quark Model [1], a meson is made of a quark q_i with one of the three colors and an antiquark $\overline{q_j}$ with the anticolor of the quark q_i. Thus, all mesons are colorless particles. Because the mesons are colorless and the intrinsic quantum numbers of the quarks are independent from the quarks' colors, we can omit the colors when we deduce the intrinsic quantum numbers of mesons from the quarks. Since we have already found the quark spectrum (Table 10 and Table 11), using sum laws, we can find the intrinsic quantum numbers (S, C, b, I and Q) of the mesons ($q_i\overline{q_j}$):

$$\begin{array}{l} S_M = S_{q_i} + S_{\overline{q_j}}, \\ C_M = C_{q_i} + C_{\overline{q_j}}, \\ b_M = b_{q_i} + b_{\overline{q_j}}, \\ Q_M = Q_{q_i} + Q_{\overline{q_j}}, \\ \overrightarrow{I}_M = \overrightarrow{I}_{q_i} + \overrightarrow{I}_{\overline{q_j}}. \end{array} \tag{58}$$

We cannot, however, find the rest masses of mesons using a sum law because of their binding energies.

A The Phenomenological Binding Energy Formula of Mesons

There is not a theoretical formula for the binding energies; thus, we propose a phenomenological formula. Because all quarks inside mesons are the excited states of the elementary ϵ-quarks, the binding energies are roughly a constant (-2Δ - 337 Mev). Quarks and antiquarks all have large rest masses. Although mesons are composed with large rest mass quarks and antiquarks, the mesons themselves do not have large masses. Thus, we use the (-2Δ) to cancel the large part (2Δ) of the quark and antiquark masses.

If the difference between the quark mass (m_i) and the antiquark mass (m_j) in the quark pair ($q_i\overline{q_j}$) is larger, the binding energy is smaller ($100\frac{\Delta m}{m_g}$). If $[(\Delta S)_i$-$(\Delta S)_j]$ is larger, the binding energy will be smaller [DS $=\left|(\Delta S)_i\text{- }(\Delta S)_j\right|$]. The pairs ($q_i\overline{q_i}$) have larger binding energies than the pairs ($q_i\overline{q_j}, i \neq j$), as shown in the following phenomenological formula for meson ($q_i\overline{q_j}$):

$$E_B(q_i\overline{q_j}) = \text{-}2\Delta \text{ -}337 + 100[\frac{\Delta m}{m_g} + DS - \widetilde{m} + \gamma(i,j) \text{ -}0.78\Delta I S_i S_j + (5.35\Delta I\text{-}2) I_i I_j] \quad (59)$$

where $\Delta = \frac{1}{3}\left|E_{bind}\right|$ (34) is $\frac{1}{3}$binding energy of a baryon (an unknown huge constant, $\Delta >> m_P = 938$ Mev); $\Delta m = |m_i\text{-}m_j|$, DS $=\left|(\Delta S)_i\text{- }(\Delta S)_j\right|$ and $\Delta I = |I_i\text{ -}I_j|$. $m_g = 939$ (Mev) unless

m_i(or m_j) equals	$m_C \geqslant 6073$	$m_b \geqslant 9333$	$m_S \geqslant 9613$
m_g will equal to	1753(Table 4)	4913 (Table7)	3753(Table 4).

(60)

$\widetilde{m} = \frac{m_i \times m_j}{m_{g_i} \times m_{g_j}}$ $\quad m_{g_i} = m_{g_j} = 939$ (Mev) unless

m_i(or m_j)	m_{q_N}=313	m_{d_s}=493	$m_{u_c} \succeq$1753	$m_{d_S}>$ 3753,	$m_{d_b} \succeq$ 4913
m_{g_j} (or m_{g_j})	313	493	1753	3753,	4913.

(61)

If q_i and q_j are both ground quarks, γ(i, j) = 0. If q_i and q_j are not both ground quarks, for $q_i = q_j$, γ(i, j) = -1; for $q_i \neq q_j$, γ(i, j) = +1. S_i (or S_j) is the strange number of the quark q_i (or q_j). I_i (or I_j) is the isospin of the quark q_i (or q_j).

We now have the sum laws (58) and binding energy formulae (59). Using these formulae, we can deduce mesons from the quark spectrum in Table 10 and Table 11.

B How to Deduce Mesons from Quarks Using these Formulae

Three types of examples show how to deduce mesons using these formulae. We will also deduce some important mesons at the same time.

B1. The Mesons of the Ground Quarks and the Ground Antiquarks

We deduce the mesons that are composed of the five ground quarks ($u^0(313)$, $d^0(313)$, $d^1_S(493)$, $u^1_C(1753)$ and $d^1_b(4913)$) and their antiquarks to show how to use the phenomenological formulae (59) to deduce the rest masses of the mesons from the quark spectrum. For the ground quarks, γ(i, j) = 0, $\widetilde{m}$ = 1 from (61) and $\Delta IS_iS_j = \Delta I(I_iI_j)$ = 0; the formula (59) is simplified to

$$E_B(q_i\overline{q_j}) = -2\Delta - 437 + 100(\frac{\Delta m}{939} + DS - 2I_iI_j). \tag{62}$$

Using (62) we deduced the masses of the important mesons as shown in Table 22:

Table 22. The Ground Mesons of the Ground Quarks

$q_i^{\Delta S}(m_i)\ \overline{q_j^{\Delta S}(m_j)}$	$\frac{100\Delta m}{939}$	DS	$2I_iI_j$	E_{bind}	Deduced	Experiment
$q^0_N(313{+}\Delta)\overline{q^0_N(313{+}\Delta)}$	0	0	$\frac{1}{2}$	- 487-2Δ	π(139)	π(138)
$q^0_N(313{+}\Delta)\overline{q^1_S(493{+}\Delta)}$	19	1	0	- 318-2Δ	K(488)	K(494)
$q^1_S(493{+}\Delta)\overline{q^1_S(493{+}\Delta)}$	0	0	0	- 437-2Δ	η(549)	η(548)
$q^1_C(1753{+}\Delta)\overline{q^0_N(313{+}\Delta)}$	153	1	0	- 184-2Δ	D(1882)	D(1869)
$q^1_C(1753{+}\Delta)\overline{q^1_S(493{+}\Delta)}$	134	0	0	- 303-2Δ	D_S(1943)	D_S(1969)
$q^1_C(1753{+}\Delta)\overline{q^1_C(1753{+}\Delta)}$	0	0	0	-437-2Δ	J/ψ(3069)	J/ψ(3097)
$q^0_N(313{+}\Delta)\overline{q^1_b(4913{+}\Delta)}$	490	1	0	153-2Δ	B(5379)	B(5279)
$q^1_S(493{+}\Delta)\overline{q^1_b(4913{+}\Delta)}$	471	0	0	34-2Δ	B_S(5440)	B_S(5370)
$q^1_C(1753{+}\Delta)\overline{q^1_b(4913{+}\Delta)}$	337	0	0	-100-2Δ	B_C(6566)	B_C(6400)
$q^1_b(4913{+}\Delta)\overline{q^1_b(4913{+}\Delta)}$	0	0	0	- 437-2Δ	Υ(9389)	Υ(9460)

From Table 22, we can see that the terms Δ of the quark and antiquark masses are always cancelled by term (-2Δ) of the binding energy. Thus we will omit the term Δ in the quark masses and the term -2Δ in the binding energy from now on.

B2. The Important Mesons of the Quark Pairs ($q_i = q_j$)

For quark pairs, $\Delta m = DS = \Delta I = 0$, γ(i, j) = -1, from (59), omitting -2Δ, we have

$$E_B(q_i\overline{q_j}) = - 437 - 100(\widetilde{m} + 2I_iI_j). \tag{63}$$

Using (63) and the sum laws (58), we deduced the mesons that are shown in Table 23 from the quarks in Table 11:

Table 23. The Important Mesons of $q(m)\overline{q(m)}$

$q_i^{\Delta s}(m)\overline{q_j^{\Delta s}(m)}$	$-100\widetilde{m}$	$-200I_iI_j$	E_{bind}	Deduced
$d_S^1(1933)\overline{d_S^1(1933)}$	$-424 \leftarrow \frac{1933\times1933}{939\times939}$	0	- 861	$\eta(3005)$
$d_S^{-1}(773)\overline{d_S^{-1}(773)}$	$-68 \leftarrow \frac{773\times773}{939\times939}$	0	-505	$\eta(1041)$
$d_S^{-1}(3753)\overline{d_S^{-1}(3753)}$	$-1597 \leftarrow \frac{3753\times3753}{939\times939}$	0	-2034	$\eta(5472)$
$d_S^0(1203)\overline{d_S^0(1203)}$	$-164 \leftarrow \frac{1203\times1203}{939\times939}$	0	-601	$\eta(1805)$
$d_S^0(1303)\overline{d_S^0(1303)}$	$-193 \leftarrow \frac{1303\times1303}{939\times939}$	0	-630	$\eta(1976)$
$d_S^0(1393)\overline{d_S^0(1393)}$	$-220 \leftarrow \frac{1393\times1393}{939\times939}$	0	-657	$\eta(2129)$
$d_S^0(2013)\overline{d_S^0(2013)}$	$-460 \leftarrow \frac{2013\times2013}{939\times939}$	0	-897	$\eta(3129)$
$d_S(2743)\overline{d_S(2743)}$	$-853 \leftarrow \frac{2743\times2743}{939\times939}$	0	-1290	$\eta(4196)$
$d_S^1(1413)\overline{d_S^1(1413)}$	$-226 \leftarrow \frac{1413\times1413}{939\times939}$	0	-663	$\eta(2163)$
$d_S^1(1513)\overline{d_S^1(1513)}$	$-260 \leftarrow \frac{1513\times1513}{939\times939}$	0	-697	$\eta(2329)$
$d_S^{-1}(1503)\overline{d_S^{-1}(1503)}$	$-256 \leftarrow \frac{1503\times1503}{939\times939}$	0	-693	$\eta(2313)$
$d_S^{-1}(1603)\overline{d_S^{-1}(1603)}$	$-291 \leftarrow \frac{1603\times1603}{939\times939}$	0	-728	$\eta(2478)$
$q_\Sigma^0(583)\overline{q_\Sigma^0(583)}$	$-39 \leftarrow \frac{583\times583}{939\times939}$	-200	-676	$\eta(490)$
$q_N^{1,0}(583)\overline{q_N^{1,0}(583)}$	$-39 \leftarrow \frac{583\times583}{939\times939}$	-50	-526	$\eta(640)$
$q_N^1(673)\overline{q_N^1(673)}$	$-51 \leftarrow \frac{673\times673}{939\times939}$	-50	-538	$\eta(808)$
$q_N^0(853)\overline{q_N^0(853)}$	$-83 \leftarrow \frac{853\times853}{939\times939}$	-50	-570	$\eta(1136)$
$q_N^0(1213)\overline{q_N^0(1213)}$	$-167 \leftarrow \frac{1213\times1213}{939\times939}$	-50	-654	$\eta(1772)$
$q_N^{1,0}(1303)\overline{q_N^{1,0}(1303)}$	$-193 \leftarrow \frac{1303\times1303}{939\times939}$	-50	-680	$\eta(1926)$
$q_N^1(1393)\overline{q_N^1(1393)}$	$-220 \leftarrow \frac{1393\times1393}{939\times939}$	-50	-707	$\eta(2079)$
$q_N^0(1573)\overline{q_N^0(1573)}$	$-281 \leftarrow \frac{1573\times1573}{939\times939}$	-50	-768	$\eta(2378)$
$q_N^{1,0}(2023)\overline{q_N^{1,0}(2023)}$	$-464 \leftarrow \frac{2023\times20233}{939\times939}$	-50	-951	$\eta(3095)$
$u_C^1(6073)\overline{u_C^1(6073)}$	$-1200 \leftarrow \frac{6073\times6073}{1753\times1753}$	0	-1637	$\psi(10509)$
$u_C^1(2133)\overline{u_C^1(2133)}$	$-148 \leftarrow \frac{2133\times2133}{1753\times1753}$	0	-585	$\psi(3681)$
$u_C^1(2333)\overline{u_C^1(2333)}$	$-177 \leftarrow \frac{2333\times2333}{1753\times1753}$	0	-614	$\psi(4052)$
$u_C^1(2533)\overline{u_C^1(2533)}$	$-209 \leftarrow \frac{2533\times2533}{1753\times1753}$	0	-646	$\psi(4420)$
$u_C^1(3543)\overline{u_C^1(3543)}$	$-408 \leftarrow \frac{3543\times3543}{1753\times1753}$	0	-845	$\psi(6241)$
$d_b^1(9333)\overline{d_b^1(9333)}$	$-361 \leftarrow \frac{9333\times9333}{4913\times4913}$	0	-798	$\Upsilon(17868)$

B3. The Mesons of the Unpaired Quarks [$q_i \neq q_j$]

For mesons ($q_i \neq q_j$), γ(i, j) = 1, from (59), omitting -2Δ,we have

$$E_B(q_i\overline{q_j}) = -237+100[\frac{\Delta m}{m_g} + DS - \widetilde{m} - 0.78\Delta I S_i S_j + (5.35\Delta I - 2) I_i I_j]. \tag{64}$$

Using the formula (64) and the sum laws (58), we deduce the mesons shown in Table 24A, Table 24B and Table 24C:

For mesons π(M) with S = C = b = 0 and I = 1, (64) can be simplified to

$$E_B(q_i\overline{q_j}) = -237+100[\tfrac{\Delta m}{m_g} + DS - \widetilde{m} - 0.78 S_i S_j + 3.35 I_i I_j]$$

Table 24-A. The Light Unflavored Mesons (S = C = b = 0 and I = 1)

$q_i(m_i)\overline{q_j(m_j)}$	$\frac{100\Delta m}{m_g}$	DS	$100\widetilde{m}$	S_iS_j	I_iI_j	E_{bind}	Deduced
$d^0_\Sigma(583)\overline{d^1_S(493)}$	9.6	1	62.1	1	0	-268	$\pi(808)$
$d^0_\Sigma(583)\overline{d^0_S(493)}$	9.6	0	62.1	1	0	-368	$\pi(708)$
$d^0_\Sigma(583)\overline{d^{-1}_S(493)}$	9.6	1	62.1	1	0	-268	$\pi(808)$
$q^0_N(313)\overline{q^0_\Delta(673)}$	38.3	0	71.7	0	$\frac{3}{4}$	-19	$\pi(967)$
$d^0_\Sigma(1033)\overline{d^1_S(493)}$	57.0	1	110.0	1	0	-268	$\pi(1258)$
$d^0_\Sigma(1033)\overline{d^0_S(493)}$	57.0	0	110.0	1	0	-368	$\pi(1158)$
$d^0_\Sigma(1033)\overline{d^{-1}_S(493)}$	57.0	1	110.0	1	0	-268	$\pi(1258)$
$q^0_N(313)\overline{q^0_\Delta(1033)}$	76.7	0	110.0	0	$\frac{3}{4}$	-19	$\pi(1327)$
$d^0_\Sigma(1303)\overline{d^1_S(493)}$	86.3	1	138.8	1	0	-268	$\pi(1528)$
$d^0_\Sigma(1303)\overline{d^0_S(493)}$	86.3	0	138.8	1	0	-368	$\pi(1428)$
$d^0_\Sigma(1303)\overline{d^{-1}_S(493)}$	86.3	1	138.8	1	0	-268	$\pi(1528)$
$q^0_N(313)\overline{q^0_\Delta(1393)}$	115.0	0	148.3	0	$\frac{3}{4}$	-19	$\pi(1687)$
$d^0_\Sigma(1753)\overline{d^1_S(493)}$	134.2	1	186.7	1	0	-268	$\pi(1978)$
$d^0_\Sigma(1753)\overline{d^0_S(493)}$	134.2	0	186.7	1	0	-168	$\pi(1878)$
$d^0_\Sigma(1753)\overline{d^{-1}_S(493)}$	134.2	1	186.7	1	0	-268	$\pi(1978)$
$q^0_N(313)\overline{q^0_\Delta(1753)}$	153.4	0	186.7	0	$\frac{3}{4}$	-19	$\pi(2077)$
$d^0_\Sigma(2023)\overline{d^1_S(493)}$	163	1	215.4	1	0	-268	$\pi(2248)$
$d^0_\Sigma(2023)\overline{d^1_S(493)}$	163	0	215.4	1	0	-368	$\pi(2148)$
$d^0_\Sigma(2023)\overline{d^1_S(493)}$	163	1	215.4	1	0	-268	$\pi(2248)$

For $\Delta IS_iS_j = \Delta I \times I_iI_j$=0, $E_B(q_i\overline{q_j})$ = - 237+100[$\frac{\Delta m}{m_g}$ +DS -$\widetilde{m}$ -2I_iI_j]

Table 24-B. The Mesons of the Unpaired Quarks (ΔIS_iS_j = $\Delta I \times I_iI_j$=0)

$q(m_i)\overline{d_S(m_j)}$	$100\frac{\Delta m}{m_g}$	DS	$2I_iI_j$	-100 $\widetilde{m}$	E_{bind}	Deduced
$q_N(313)\ \overline{q_N^0(583)}$	29 ←($\frac{270}{939}$)	0	$\frac{1}{2}$	62← $\frac{313\times583}{313\times939}$	-320	$\eta(576)$
$q_N(313)\overline{q_N^1(583)}$	29 ← $\frac{270}{939}$	1	$\frac{1}{2}$	62← $\frac{313\times583}{313\times939}$	-220	$\eta(676)$
$q_N(313)\overline{q_N^1(673)}$	38 .3← $\frac{360}{939}$	1	$\frac{1}{2}$	71.7← $\frac{313\times673}{313\times939}$	-220	$\eta(766)$
$q_S^1(493)\overline{d_S^{-1}(773)}$	30← $\frac{280}{939}$	2	0	82.3← $\frac{493\times773}{493\times939}$	- 90	$\eta(1177)$
$q_N(313)\overline{q_N^0(1303)}$	105.4 ← $\frac{990}{939}$	0	$\frac{1}{2}$	138.8← $\frac{313\times1303}{313\times939}$	-320	$\eta(1296)$
$q_N(313)\overline{q_N^1(1303)}$	105.4 ← $\frac{990}{939}$	1	$\frac{1}{2}$	138.8← $\frac{313\times1303}{313\times939}$	-220	$\eta(1396)$
$q_S^1(493)\overline{d_S^1(1513)}$	108.6← $\frac{920}{939}$	0	0	161.1← $\frac{493\times1513}{493\times939}$	-290	$\eta(1716)$
$q_S^1(493)\overline{d_S^{-1}(1603)}$	118.2← $\frac{1110}{939}$	2	0	170.7← $\frac{493\times1603}{493\times939}$	-90	$\eta(2006)$
$q_S(493)\overline{d_S(3753)}$	347.2← $\frac{3260}{939}$	2	0	399.7← $\frac{493\times3753}{493\times939}$	-90	$\eta(4156)$
$q_N(313)\overline{d_S^{\pm}(773)}$	49 ← $\frac{460}{939}$	1	0	82.3← $\frac{313\times773}{313\times939}$	- 170	K(916)
$q_N(313)\overline{d_S^0(773)}$	49 ← $\frac{460}{939}$	0	0	82.3← $\frac{313\times773}{313\times939}$	-270	K(816)
$q_N(313)\overline{d_S^0(1203)}$	94.8 ← $\frac{890}{939}$	0	0	128.1← $\frac{313\times1203}{313\times939}$	-270	K(1246)
$q_N(313)\overline{d_S^0(1303)}$	105.4← $\frac{990}{939}$	0	0	138.8← $\frac{313\times1303}{313\times939}$	-270	K(1346)
$q_N(313)\overline{d_S^0(1393)}$	115 ← $\frac{1080}{939}$	0	0	148.3← $\frac{313\times1393}{313\times939}$	-270	K(1436)
$q_N(313)\overline{d_S^{1}(1513)}$	126.7← $\frac{1190}{939}$	1	0	160← $\frac{313\times1503}{313\times939}$	-170	K(1646)
$q_N(313)\overline{d_S^1(1933)}$	172.5← $\frac{1620}{939}$	1	0	205.9← $\frac{313\times1933}{313\times939}$	-170	K(2076)
$u_C(6073)\overline{q_N(313)}$	328,6← $\frac{5760}{1753}$	1	0	346.4← $\frac{313\times6073}{313\times1753}$	-155	D(6231)
$u_C^1(1753)\overline{d_S^{-1}(773)}$	104.4← $\frac{980}{939}$	2	0	82.3← $\frac{1753\times773}{1753\times939}$	-15	$D_S(2511)$
$u_C^1(1753)\overline{d_S^0(773)}$	104.4← $\frac{980}{939}$	1	0	82.3← $\frac{1753\times773}{1753\times939}$	-115	$D_S(2411)$
$u_C^1(1753)\overline{d_S^1(773)}$	104.4← $\frac{980}{939}$	0	0	82.3← $\frac{1753\times773}{1753\times939}$	-215	$D_S(2311)$
$q_N(313)\overline{d_b^1(9333)}$	183.6← $\frac{9020}{4913}$	1	0	190← $\frac{313\times9333}{313\times4913}$	-143	B(9503)
$d_S^{-1}(493)\overline{d_S^{-1}(9613)}$	243← $\frac{9120}{3753}$	0	0	256.1← $\frac{493\times9613}{493\times3753}$	-250	$\eta(9856)$
$d_S^1(493)\overline{d_S^{-1}(9613)}$	243← $\frac{9120}{3753}$	2	0	256.1← $\frac{493\times9613}{493\times3753}$	-50	$\eta(10056)$
$d_S^{-1}(773)\overline{d_S^{-1}(9613)}$	235.5← $\frac{8840}{3753}$	0	0	210.9← $\frac{773\times9613}{939\times3753}$	-212	$\eta(10174)$
$d_S^0(1203)\overline{d_S^{-1}(9613)}$	224← $\frac{8410}{3753}$	1	0	328.2← $\frac{1203\times9613}{939\times3753}$	-241	$\eta(10575)$
$d_S^1(1413)\overline{d_S^{-1}(9613)}$	218.5← $\frac{8200}{3753}$	2	0	385← $\frac{1413\times9613}{939\times3753}$	-204	$\eta(10822)$
$d_S^1(1513)\overline{d_S^{-1}(9613)}$	215.8← $\frac{8100}{3753}$	2	0	412.7← $\frac{1513\times9613}{939\times3753}$	-234	$\eta(10892)$
$d_S^0(1923)\overline{d_S^{-1}(9613)}$	204.9← $\frac{7690}{3753}$	1	0	524.5← $\frac{1923\times9613}{939\times3753}$	-456	$\eta(11080)$
$d_S^1(1933)\overline{d_S^{-1}(9613)}$	204.6← $\frac{7680}{3753}$	2	0	527.3← $\frac{1933\times9613}{939\times3753}$	-360	$\eta(11186)$

For $S_iS_j = I_iI_j = 0$, $E_B(q_i\overline{q_j})$ = -237+100[$\frac{\Delta m}{m_g}$ +DS -$\widetilde{m}$].

Table 24-C. The Mesons with $S_iS_j = I_iI_j = 0$

$q_i(m_i)\overline{q_j(m_j)}$	$\frac{100\Delta m}{m_g}$	DS	$100\widetilde{m}$	E_{bind}	Deduced
$u_C^1(1753)\overline{u_C^1(2133)}$	40.5	0	121.7	-318	$\psi(3568)$
$u_C^1(1753)\overline{u_C^1(2333)}$	61.8	0	133.1	-308	$\psi(3778)$
$u_C^1(1753)\overline{u_C^1(2533)}$	83.1	0	144.5	-298	$\psi(3988)$
$q_N^0(313)\overline{u_C^1(2133)}$	193.8	1	121.7	-65	D(2381)
$q_N^0(313)\overline{u_C^1(2333)}$	215.1	1	133.1	-55	D(2591)
$q_N^0(313)\overline{u_C^1(2533)}$	236.4	1	144.5	-45	D(2801)
$u_C^1(2133)\overline{d_S^1(493)}$	174.7	0	121.7	-184	$D_S(2442)$
$u_C^1(2333)\overline{d_S^1(493)}$	196.0	0	133.1	-174	$D_S(2652)$
$u_C^1(2533)\overline{d_S^1(493)}$	217.3	0	144.5	-164	$D_S(2862)$
$q_N^0(583)\overline{q_b^1(4913)}$	461.1	1	62.1	+262*	B(5758)
$q_N^1(583)\overline{q_b^1(4913)}$	461.1	0	62.1	+162*	B(5658)
$q_N^0(853)\overline{q_b^1(4913)}$	432.4	1	90.8	+205*	B(5971)
$d_S^{-1}(773)\overline{q_b^1(4913)}$	440.9	2	82.3	+322*	$B_S(6008)$
$d_S^0(773)\overline{q_b^1(4913)}$	440.9	1	82.3	+222*	$B_S(5908)$
$d_S^1(773)\overline{q_b^1(4913)}$	440.9	0	82.3	+122*	$B_S(5808)$

* $E_B(q_i\overline{q_j})$ = -2Δ -337+..., Δ >> 337+.... Thus $E_B(q_i\overline{q_j}) < 0$.

As in Table 22, Table 23 and Table 24, using the sum laws (58) and the binding energy formula (59), we deduced a meson spectrum from the quark spectrum (Table 10 and Table 11). Table 25-Table 31 will show the results. At the same time, we will compare the deduced result with the experimental results using Table 25-Table 31 also in the following section.

C Comparing with the Experimental Results of Mesons

Using Table 25-Table 31, we compare the deduced meson spectrum with the experimental results [12]. In the comparison, we do not take into account the angular momenta of the experimental results. We assume that the small differences of the masses in the

same group of mesons with the same intrinsic quantum numbers are from their different angular momenta. If we ignore this effect, their masses would be essentially the same. In this comparison, we use the meson name to represent the intrinsic quantum numbers. If the names of the deduced meson and the experimental meson are the same, this means that the intrinsic quantum numbers (I, S, C, b and Q) are exactly the same. Since the unflavored mesons with the same intrinsic quantum numbers but different angular momenta and parities have different names, in order to compare the rest masses, we omit the differences of the angular momenta and parities; we use meson η to represent the mesons with S = C = b = 0, I = Q = 0 (η, ϖ, ϕ, h and f) (see Table C5 and Table 27) and we use meson π to represent the mesons with S = C = b = 0, I = 1, Q = 1, 0, -1 (π, ρ, a and b) (see Table C6 and Table 28). Table 25-Table 31 show that the deduced intrinsic quantum umbers (I, S, C, b and Q) exactly match with the experimental results.

Sometimes there are multiple possible mesons with the same intrinsic quantum numbers and the same (or nearly same) rest masses, we will choose the meson with the highest probability using (47). If there are several possible mesons that have essentially the same probability, it is believed that they form (superposition) one meson with the average mass of all the mesons (see Table C3-Table C7).

Using Table 25-Table 31, we compare the deduced mesons with the experimental results. These tables show that although the names are not the same, the intrinsic quantum numbers (I, S, C,b and Q) are the same for both deduced meson and experimental meson. We do not repeat this conclusion for each table. Their mass units are the same-“Mev”.

We compare the heavy unflavored mesons in Table 25:

Table 25. The Heavy Unflavored Mesons with S=C=b=I=0

$d_S^{-1}(9613)\overline{d_S(m)}$	Deduced	Expert., Γ (Mev)	R=$\frac{\Delta M}{M}$%
$d_b(4913)\overline{d_b(4913)}$	$\Upsilon(9389)$	$\Upsilon(9460)$, 53 kev	0.75
$d_S^{-1}(9613)\overline{d_S^0(493)}$	$\eta(\overline{9906})$	$\chi(\overline{9888})$, ?	0.18
$d_S^{-1}(9613)\overline{d_S^1(493)}$	$\eta(10056)$	$\Upsilon(10023)$, 43 kev	0.13
$d_S^{-1}(9613)\overline{d_S^{-1}(773)}$	$\eta(\overline{10174})$	$\chi(\overline{10251})$, ?	0.75
$u_C(6073)\overline{u_C(6073)}$	$\psi(10509)$	$\Upsilon(10355)$, 26 kev	1.5
$2d_S^{-1}(9613)\overline{d_S^0(1203)_F}$	$\eta(\overline{10620})$	$\Upsilon(10580)$, 20	0.66
$d_S^{-1}(9613)\overline{d_S^{-1}(1603)_F}$	$\eta(\overline{10777})$	$\Upsilon(10865)$, 110	0.73
$d_S^{-1}(9613)\overline{d_S^0(1923)_F}$	$\eta(11079)$	$\Upsilon(11020)$, 79	0.54
$d_S^{-1}(9613)\overline{d_S^0(2013)}$	$\eta(11151)$	Prediction	
$d_S^{-1}(9613)\overline{d_S^{-1}(3753)}$	$\eta(12261)$	Prediction	

Table 25 shows that the masses of the deduced mesons are more than 98.5% consistent with the experimental results.

We compare the intermediate mass mesons in Table 26:

Table 26. The Intermediate Mass Mesons (S=C=b=I=0)

$q_i(m)\overline{q_i(m)}$	Deduced	Exper., Γ (Mev)	R=$\frac{\Delta M}{M}$%
$d_S^1(1933)\overline{d_S^1(1933)}$	$\eta(3005)$	$\eta_C(2980)$, 17	0.84
$q_C^1(1753)\overline{q_C^1(1753)}$	$J/\psi(3069)$	$J/\psi(3097)$, 91 kev	0.91
$q_C^1(1753)\overline{u_C^1(2133)}$	$\psi(3567)$	$\chi(\overline{3494})$, $\overline{2}$	2.09
$u_C^1(2133)\overline{u_C^1(2133)}$	$\psi(3681)$	$\psi(3686)$, 281 kev	0.14
$q_C^1(1753)\overline{u_C^1(2333)}$	$\psi(3778)$	$\psi(3770)$, 23.6	0.21
$u_C^1(2333)\overline{u_C^1(2333)}$	$\psi(4052)$	$\psi(4040)$, 52	0.11
$d_S^{-1}(3753)\overline{d_S^1(493)}$	$\eta(4156)$	$\psi(4160)$, 78	0.10
$u_C^1(2533)\overline{u_C^1(2533)}$	$\psi(4420)$	$\psi(4415)$, 43	0.30
$q_C^1(1753)\overline{u_C^1(3543)}$	$\psi(4959)$	Prediction	

Table 26 shows that the masses of the deduced mesons are about 98% consistent with the experimental results.

We compare the light unflavored mesons with I = 0 in Table 27:

Table 27. The Light Unflavored Mesons (S=C=b=0) I = 0

$q_i(m_i)\overline{q_j(m_j)}$	Deduced	Exper., Γ (Mev)	$R=\frac{\Delta M}{M}\%$
$q_S^1(493)\overline{q_S^1(493)}$	$\eta(549)$	$\eta(547)$, 1.18 Kev	0.4
$q_N^0(313)\overline{q_N^0(583)}$	$\eta(\overline{584})$	$\eta(600)$, 800	/
$q_N^0(313)\overline{q_N^1(673)}$	$\eta(\overline{774})$	$\varpi(782)$, 8.49	1.15
$d_S^1(493)\overline{d_S^1(773)}$	$\eta(976)$	$\eta(\overline{969}),\overline{35}$	0.93
$d_S^{-1}(773)\overline{d_S^{-1}(773)}$	$\eta(1041)$	$\phi(1020)$, 4.26	2.1
$q_N^0(313)\overline{q_N^0(1213)}$	$\eta(\overline{1166})$	$h_1(1170)$, 360	0.26
$q_N^0(313)\overline{q_N^0(1303)}$	$\eta(1296)$	$\eta(\overline{1284}),\overline{88}$	0.94
$q_N^0(313)\overline{q_N^1(1303)}$	$\eta(1396\ )$	$\eta(\overline{1403}),\overline{147}$	0.50
$q_N^0(313)\overline{q_N^1(1393)}$	$\eta(\overline{1526})$	$\eta(\overline{1503})$, $\overline{90}$	1.53
$d_S^1(493)\overline{d_S^0(1303)}$ $d_S^1(493)\overline{d_S^0(1393)}$	$\eta(\overline{1666})$	$\eta(\overline{1670}),\overline{191}$	0.18
$d_S^1(493)\overline{d_S^1(1513)}$ $d_S^1(493)\overline{d_S^{-1}(1503)}$	$\eta(\overline{1870})$	$\phi(\overline{1899}),\overline{125}$	1.53
$q_N^0(313)\overline{q_N^0(2023)}$	$\eta(2016)$	$\eta(\overline{2011})$, $\overline{259}$	0.25
$4q_N^0(313)\overline{q_N^1(2023)}$	$\eta(\overline{2116})$	$\eta(\overline{2130})^*$, $\overline{187}$	0.66
$q_N^0(313)\overline{q_N^0(2293)}$	$\eta(2216)$	$\eta(\overline{2218})^*$, $\overline{109}$	0.09
$d_S^1(493)\overline{d_S^0(2013)}$	$\eta(2316)$	$\eta(\overline{2318})^*$, $\overline{234}$	0.09
$d_S^{-1}(1603)\overline{d_S^{-1}(1603)}$	$\eta(\overline{2428})$	$f_0(2465)^*$, 255	1.5
$q_N^0(313)\overline{q_N^0(2653)}$	$\eta(2646)$	Prediction	
$q_S^1(493)\overline{d_S^0(2743)}$	$\eta(3046)$	Prediction	

Table 27 shows that the masses of the deduced mesons are about 98% consistent with the experimental results.

We compare the light unflavored mesons with I = 1 in Table 28:

Table 28. The Light Unflavored Mesons (S=C=b=0) I = 1

$q_N(313)\overline{q_j(m_j)}$	E_{bind}	Phenomen.	Exper., Γ (Mev)	$R=\frac{\Delta M}{M}\%$
$q_N^0(313)\overline{q_N^0(313)}$	-487	$\pi(139)$	$\pi(138)$	0.72
$d_S^{\pm1,0}(493)\overline{d_\Sigma^0(583)}$	$\overline{-300}$	$\pi(\overline{775})$	$\pi(776)$, 150	0.0
$q_N^0(313)\overline{q_\Delta^0(673)}$	-19	$\pi(967)$	$a_0(985)$, 75	1.8
$d_S^{\pm1,0}(493)\overline{d_\Sigma^0(1033)}$	$\overline{-300}$	$\pi(\overline{1225})$	$\pi(\overline{1230})$,$\overline{287}$	0.24
$q_N^0(313)\overline{q_\Delta(1033)}$	-19	$\pi(1327)$	$\pi(\overline{1331})$, $\overline{269}$	0.30
$d_S^{\pm1,0}(493)\overline{d_\Sigma(1303)}$	$\overline{-300}$	$\pi(\overline{1495})$	$\pi(\overline{1470})$,$\overline{323}$	1.8
$q_N^0(313)\overline{q_\Delta(1393)}$	-19	$\pi(1687)$	$\pi(1669)$, $\overline{246}$	1.1
$d_S^{\pm1,0}(493)\overline{d_\Sigma(1753)}$	$\overline{-300}$	$\pi(\overline{1825})$	$\pi(1812)$,207	0.77
$q_N^0(313)\overline{q_\Delta(1753)}$	-19	$\pi(\overline{2001})$	$\pi(\overline{2010})$, $\overline{353}$	0.40
$d_S^{\pm1,0}(493)\overline{d_\Sigma(2023)}$	-268	$\pi(\overline{2248})$	$\pi(\overline{2250})^*$, $\overline{281}$	0.09
$q_N^0(313)\overline{q_\Delta(2113)}$	-19	$\pi(2407)$	$\pi(\overline{2450})^*$, $\overline{400}$	1.8
$d_S^{\pm1,0}(493)\overline{d_\Sigma(2473)}$	$\overline{-300}$	$\pi(2666)$	Prediction	

Table 28 shows that the masses of the deduced mesons are more than 98% consistent with the experimental results.

We compare the mesons B and B_S in Table 29:

Table 29. The Mesons B(M) and B_S(M)

$d_b(m)\overline{q_N(m_j)}$	Deduced	Exper., (Mev)	$R=\frac{\Delta M}{M}\%$
$q_N^0(313)\overline{q_b^1(4913)}$	B(5378)	$B(\overline{5302})^{\#}$	1.4
$q_N^{1,0}(583)\overline{q_b^1(4913)}$	B(5708)	$B_j^*(5732)$	0.42
$q_N^0(853)\overline{q_b^1(4913)}$	B(5971)	Prediction	
$q_N^0(313)\overline{d_b^1(9333)}$	B(9503)	Prediction	
$q_S^1(493)\overline{q_b^1(4913)}$	$B_S(5440)$	$B_S(5370)$	1.3
$q_b^1(4913)\overline{d_S(773)}$	$B_S(5908)$	$B_{SJ}^*(5850)$	1.0
$q_S(493)\overline{q_b(9333)}$	$B_S(9579)$	Prediction	

$B(\overline{5302})^{\#}=\frac{1}{2}[B(5279)+B(5325)]$

Table 29 shows that the masses of the deduced mesons are about 98% consistent with the

experimental results.

Table 30. The D-mesons and the D_S-Mesons

$u_C(m)\overline{q^0_N(313)}$	Deduced	Exper.,(Mev)	$R=\frac{\Delta M}{M}\%$
$u_C(1753)\overline{q^0_N(313)}$	D(1882)	D($\overline{1939}$)	2.9
$u^1_C(2133)\overline{q^0_N(313)}$	D(2381)	D($\overline{2440}$)	2.4
$u^1_C(2333)\overline{q^0_N(313)}$	D(2591)	D(2640)*	1.9
$u^1_C(2533)\overline{q^0_N(313)}$	D(2801)	Prediction	
$u^1_C(6073)\overline{q^0_N(313)}$	D(6231)	Prediction	
****************	*******	********	***
$u^1_C(1753)\overline{d^1_S(493)}$	$D_S(1943)$	$D^{\pm}_S(1968)$	1.3
$u^1_C(1753)\overline{d_S(493)}$	$D_S(\overline{2093})$	$D_S(\overline{2112})$	0.90
$u^1_C(2133)\overline{d^1_S(773)}$	$D_S(2311)$	$D_{S_j}(2317)$	0.04
$u^1_C(2133)\overline{d^1_S(493)}$	$D_S(\overline{2455})$	$D_{S_j}(2460)$	0.20
$u^1_C(2333)\overline{d^1_S(493)}$	$D_S(\overline{2597})$	$D_{S_j}(\overline{2555})$	0.23
$u^1_C(2333)\overline{d^1_S(493)}$	$D_S(\overline{2761})$	Prediction	

Table 31. The Strange Mesons (S = ± 1)

$q_N(313)\ \overline{d_S(m)}$	Deduced	Exper., Γ (Mev)	$R=\frac{\Delta M}{M}\%$
$q_N(313)\overline{d_S(493)}$	K(488)	K(494)	1.2
$q_N(313)\overline{d_S(773)}$	K($\overline{883}$)	K(892), 50	1.3
$q_N(313)\overline{d^0_S(1203)}$	K($\overline{1279}$)	K(1273), 90	0.47
$q_N(313)\overline{d^0_S(1393)}$	K($\overline{1436}$)	K($\overline{1414}$), $\overline{200}$	1.6
$q_N(313)\overline{d^{-1}_S(1503)}$	K($\overline{1606}$)	K($\overline{1615}$)$^{\#}$, $\overline{92}$	0.50
$q_N(313)\overline{d^{-1}_S(1603)}$	K($\overline{1766}$)	K($\overline{1771}$), $\overline{236}$	0.28
$q_N(313)\overline{d^0_S(1923)}$	K(1966)	K($\overline{1962}$)$^{\#}$, $\overline{287}$	0.20
$q_N(313)\overline{d^0_S(2013)}$	K(2056)	$K^*_4(2045)$, 198	1.0
$q^0_N(2013)\overline{d^1_S(493)}$	K(2316)	K($\overline{2317}$)$^{\#}$, $\overline{170}$	0.04
$q_N(313)\overline{d^0_S(2743)}$	K(2786)	Prediction	?

Table 30 and Table 31 show that the deduced intrinsic quantum numbers of the D-mesons, the D_S-mesons and the K-mesons are the same as the experimental results. They also show

that the deduced rest masses of the D-mesons, the D_S-mesons and the K-mesons agree (about 98%) with the experimental results.

In summary, the phenomenological formulae explain all meson experimental intrinsic quantum numbers (100%) and the rest masses (about 98%). We explain virtually all experimentally confirmed mesons in this paper and all deduced low-mass mesons have already been discovered by experiments.

VI Evidence for the Deduced Quarks

According to the Quark Model [1], we cannot see an individual quark. We can only infer the existence of quarks from the existence of baryons and mesons, which are made up of the quarks. Thus, if we find the mesons that are made of certain quarks, it means that we also find the quarks. For example, from meson J/Ψ(3097) = [u_C^1(1753)$\overline{u_C^1(1753)}$], we discovered the u_C^1(1753)-quark. Similarly the experimental baryon spectrum [11] and the meson spectrum [12] have already provided evidence of almost all of the quarks in Table 11. As examples, for the three three-brother-quarks [d_S^{-1}(773), d_S^{-1}(3753) and d_S^{-1}(9613)], [u(313), u_C^1(1753) and u_C^1(6073)] and [u_C^1(2133), u_C^1(2333) and u_C^1(2533)] and a four-brother-quark [d(313), d_S^1(493), d_S^1(1933) and d_b^1(4913)], they all have strong experimental evidence.

A The Three Brother Quarks d_S^{-1}(773), d_S^{-1}(3753) and d_S^{-1}(9613)

There are three brother quarks, d_S^{-1}(773), d_S^{-1}(3753) and d_S^{-1}(9613). They are born on the single energy bands of the Δ-axis at the symmetry point H from ΔS = -1 (see Table 4). From Table 13, Table 25 and Table 26, we have:

A1. The d_S^{-1}(773)-quark (see Table 4)

The d_S^{-1}(773)-quark has Q = -$\frac{1}{3}$, n =(0, 0, 2). Table 32 shows its evidence:

Table 32. Evidence of the $d_S^{-1}(773)$-quark

where	The Quark Constitutions of the Hadrons	Exper.	$\frac{\Delta M}{M}\%$
Table 17	$d_S^{-1}(773)u(313)d(313) = \Lambda(1399)$	$\Lambda(1405)$	0.35
	$d_S^{-1}(773)u(313)u(313) = \Sigma^+(1399)$	$\Sigma^+(1383)$	1.13
	$d_S^{-1}(773)u(313)d(313) = \Sigma^0(1399)$	$\Sigma^0(1384)$	1.0
	$d_S^{-1}(773)d(313)d(313) = \Sigma^-(1399)$	$\Sigma^-(1387)$	0.87
Table 27	$d_S^{-1}(773)\overline{d_S^{-1}(773)} = \eta(1041)$	$\phi(1020)$	2.1
Table 31	$q_N(313)\overline{d_S^{-1}(773)}$ =K(883)	K(892)	1.3
Table 24B	$d_S(493)\overline{d_S^{-1}(773)} = \eta(1177)$	$h_1(1170)$	0.1
Table 24B	$u_C(1753)\overline{d_S^{-1}(773)} = D_S(2511)$	$D_{S_1}(2536)$	1.0

There is enough evidence to show that the $d_S^{-1}(773)$-quark really does exist.

A2. The $d_S^{-1}(3753)$-quark (Table 4 and Table 26)

The $d_S^{-1}(3753)$-quark has Q = $-\frac{1}{3}$, n =(0, 0, 4). Table 33 show its evidence:

Table 33 Evidence of the $d_S^{-1}(3753)$-quark (see Table 26)

$q_i(m)\overline{q_i(m)}$	DS	$\frac{100\Delta m}{939}$	$-100\widetilde{m}$	E_{bind}	Theory	Exper.	$\frac{\Delta M}{M}\%$
$d_S^{-1}(3753)\overline{d_S^{1}(493)_\Sigma}$	2	347	-400	-90	$\eta(4156)$	$\psi(4160$	.093

A3. The $d_S^{-1}(9613)$-quark (Table 4 and Table 25)

The $d_S^{-1}(9613)$-quark has Q = $-\frac{1}{3}$, n =(0, 0, 6). Table 34 show its evidence:

Table 34. Evidence of the $d_S^{-1}(9613)$-quark (see Table 25)

$d_S^{-1}(9613)\overline{d_S(m)}$	E_{bind}	Theory	Exper., Γ (Mev)	R.=$\frac{\Delta m}{m}\%$
$d_S^{-1}(9613)\overline{d_S^{-1}(493)_D}$	-250	$\eta(\overline{9906})$	$\chi(\overline{9889})$, ?	0.17
$d_S^{-1}(9613)\overline{d_S^{1}(493)_\Sigma}$	-50	$\eta(10056)$	$\Upsilon(10023)$, 0.044	0.13
$d_S^{-1}(9613)\overline{d_S^{-1}(773)_\Delta}$	- 212	$\eta(\overline{10174})$	$\chi(\overline{10252})$, ?	0.76
$d_S^{-1}(9613)\overline{d_S^{0}(1303)_F}$	- 247	$\eta(\overline{10620})$	$\Upsilon(10580)$, 14	0.66
$d_S^{-1}(9613)\overline{d_S^{-1}(1603)_F}$	- 347	$\eta(\overline{10777})$	$\Upsilon(10860)$, 110	0.14
$d_S^{-1}(9613)\overline{d_S^{0}(1923)_F}$	-455	$\eta(11079)$	$\Upsilon(11020)$, 79	0.61

Thus, the three brother quarks [$d_S^{-1}(773)$, $d_S^{-1}(3753)$ and $d_S^{-1}(9613)$] really do exist.

B The Three Brother Quarks u_C^1(2133), u_C^1(2333) and u_C^1(2533)

There are three brother quarks u_C^1(2133), u_C^1(2333) and u_C^1(2533) with I = 0 and Q = $\frac{2}{3}$ on the D-axis. They are born from ΔS = +1 on the twofold bands with two inequivalent n-values of the D-axis (see Table B5). From Table 14, Table 20, Table 26 and Table 30, we can find:

B1. The u_C^1(2133)-quark (see Table B5, Table 26 and Table C7)

The u_C(2133)-quark has Q = $\frac{2}{3}$. Table 35 shows its evidence:

Table 35. The Evidence of the u(313)-Quark

Deduced Baryon or Mesons	experiment	$\frac{\Delta M}{M}\%$
u_C^1(2133)u(313)d(313)= Λ_C^+(2609)	$\Lambda_C^+(\overline{2611})$	0.08
$\mathrm{u}_C^1(2133)\overline{\mathrm{u}_C^1(2133)}$ = ψ(3681)	ψ(3686),	0.14
$\mathrm{q}_N(313)\overline{\mathrm{u}_C^1(2133)}$ = D(2381)	D$(\overline{2440})$,	2.42
$\mathrm{u}_C^1(2133)\overline{\mathrm{d}_S^1(493)}$ = D_S(2442)	D_{S_j}(2460)	1.90

B2. The u_C(2333)-quark (Table B5 and Table 26)

The u_C(2333)-quark has Q = $\frac{2}{3}$. Table 36 shows its evidence:

Table 36. The Evidence of the u(313)-Quark

Deduced Baryons or Mesons	Experiments	$\frac{\Delta M}{M}\%$
u_C^1(2333)u(313)d(313)= Λ_C^+(2809)	$\Lambda_C^+(\overline{2823})$	0.50
$\mathrm{u}_C^1(2333)\overline{\mathrm{u}_C^1(2333)}$ = ψ(4052)	ψ(4040))	0.30
$\mathrm{q}_N(313)\overline{\mathrm{u}_C^1(2333)}$ = D(2591)	D(2640)*	1.90

B3. The u_C^1(2533)-quark (Table B5 and Table 26)

The u_C^1(2533)-quark has Q = $\frac{2}{3}$. Table 37 shows its evidence:

Table 37. The Evidence of the u(313)-Quark

Deduced Baryon or Mesons	Experiments	$\frac{\Delta M}{M}\%$
$\mathrm{u}_C^1(2533)\overline{\mathrm{u}_C^1(2533)}$ = ψ(4420)	ψ(4415), Γ = 43	0.11

Table 35-Table 37 show that the three brother quarks u_C(2133), u_C(2333) and u_C(2533) do exist.

C The Three Brother Quarks u(313), $u_C^1(1753)$ and $u_C^1(6073)$

There are three brother quarks u(313), $u_C^1(1753)$ and $u_C^1(6073)$ also. They are born at the Γ-point of the single energy bands of the Δ-axis (table 4). With Table 16, Table 22, Table 25 and Table 26, we get:

C1. The u(313)-quark (Table 4)

The u(313)-quark has Q = $\frac{2}{3}$, $\overrightarrow{n}$ = (0, 0, 0). Table 38 shows its evidence:

Table 38. The Evidence of the u(313)-Quark

Deduced Baryon or Mesons	Experiments
u(313)u(313)d(313)=p(939)	P(938), τ= 2.1×10^{29}years
u(313)$\overline{u(313)}$=π^0(139)	π^0(135), τ= (8.4 ± 0.6) × 10^{-17} s
q_N(313)$\overline{u(313)}$= ⟨ π^0(139) / π^-(139)	π^0(135), τ= (8.4 ± 0.6) × 10^{-17} s π^-(140), τ= (2.6033 ± 0.0005) × 10^{-8} s
u(313)$\overline{d_S^1(493)}$= K^+(488)	K^+(494), (1.2384 ± 0.0024) × 10^{-8} s

C2. The $u_C^1(1753)$-quark (Table 4)

The $u_C^1(1753)$-quark has Q = $\frac{2}{3}$, $\overrightarrow{n}$ = (0, 0, -2). Table 39 shows its evidence:

Table 39. The Evidence of the $u_C^1(1753)$-Quark

Deduced Baryon or Mesons	Experiments
u_C^1(1753)u(313)d(313)= Λ_C^+(2279)	Λ_C^+(2285), τ= (200 ± 6) × 10^{-15} s
u_C^1(1753)$\overline{u_C^1(1753)}$ = ψ(3069)	J/ψ(3097), Γ = 91 ± 3.2 kev
q_N(313)$\overline{u_C^1(1753)}$= D(1882)	D(1869), τ= (1040 ± 7) × 10^{-15} s
u_C^1(1753)$\overline{d_S(493)}$= D_S(1943)	D_S(1968), τ= (490 ± 9) × 10^{-15} s

C3. The $u_C^1(6073)$-quark (see Table4 and Table 25)

The $u_C^1(6073)$-quark has Q = $\frac{2}{3}$, $\overrightarrow{n}$ = (0, 0, -4). Table 40 shows its evidence:

Table 40. The Evidence of the u(6073)-quark

q(m)$\overline{q(m)}$	-100$\widetilde{m}$	I_iI_j		Deduced	Exper., Γ (kev)
u_C^1(6073)$\overline{u_C^1(6073)}$	-1200← $\frac{6073\times6073}{1753\times1753}$	0	-1637	ψ(10509)	Υ(10355), 26

Table 40 shows very strong evidence of the $u_C^1(6073)$-quark: 1) ψ(10509) and Υ(10355) have exactly the same intrinsic quantum numbers (I = S = C = b = Q = 0), 2) they have essentially the same rest masses, and 3) the width Γ = 26 kev of Υ(10355) < the

width Γ = 87 kev of J/ψ(3067). It is well known that the discovery of J/ψ(3067) is also the discovery of the u_C^1(1753)-quark. Thus, the discovery of Υ(10355) [ψ(10509)] is also the discovery of the u_C^1(6073)-quark.

Table 38-Table 40 shows that the three brother quarks really do exist.

D The Four Brother Quarks d(313), d_S^1(493), d_S^1(1933) and d_b^1(4913)

From Table 7, we see the following four brother quarks: at E_Γ= 0, $\overrightarrow{n}$ =(0,0,0), d(313); at E_N=1/2, $\overrightarrow{n}$ = (1,1,0), d_S^1(493); at E_N=9/2, $\overrightarrow{n}$=(2,2,0), d_S^1(1933); and at E_N=25/2, $\overrightarrow{n}$=(3,3,0), d_b^1(4913). They are born on the single energy bands of the Σ-axis. The four "brothers" have the same electric charge (Q = -1/3) and the same generalized strange number (S_G = S + C + b = -1). Using Table 7, Table 22, Table 26, Table 28 and Table 30, we find:

D1. The d(313)-Quark (Table 7)

The d(313)-Quark has Q = -$\frac{1}{3}$, $\overrightarrow{n}$= (0, 0, 0). Table 41 show its evidence:

Table 41. The Evidence of the d(313)-Quark

d(313)u(313)d(313)=n(939)	n(940), τ=(885.7$\pm$ 0.8)s
d(313)$\overline{\text{d(313)}}$=π^0(139)	π^0(135), τ=(2.6033 $\pm$ 0.0005)$\times$ 10^{-8}s
q_N(313)$\overline{\text{d(313)}}$= $\langle$ π^+(139) π^0(139)	π^+(139), τ=(2.6033 $\pm$ 0.0005)$\times$ 10^{-8}s π^0(135), τ= (8.4 $\pm$ 0.6) $\times$ 10^{-17} s
d_S^1(493)$\overline{\text{d(313)}}$= K^0(488)	K^0(494), τ= (1.2384 $\pm$ 0.0024) $\times$ 10^{-8} s

D2. The d_S^1(493)-quark (Table 7)

The d_S^1(493)-Quark has Q = -$\frac{1}{3}$, $\overrightarrow{n}$ =(1, 1, 0). Table 42 shows its evidence:

Table 42. Evidence of the d_S^1(493)-quark

d_S^1(493)u(313)d(313)=Λ(1119)	Λ(1116), τ=(2.632$\pm$ 0.020)$\times$ 10^{-10}s
d_S^1(493)$\overline{d_S^1\text{(493)}}$=$\eta$(549)	η(548), Γ=(1.29 $\pm$ 0.11) kev
q_N(313)$\overline{d_S^1\text{(493)}}$= K(488)	K(494), Γ = (1.2384 $\pm$ 0.0024) $\times$ 10^{-8}s

D3. The d^1_S(1933)-quark (Table 7)

The d^1_S(1933)-quark has Q = -$\frac{1}{3}$, $\overrightarrow{n}$ =(2, 2, 0). Table 43 shows its evidence:

Table 43. The Evidence of the d_S(1933)-Quarks

$d_S(1933)u(313)d(313)=\Lambda(2559)$	$\Lambda(2585)$, τ=?;
$d^1_S(1933)\overline{d^1_S(1933)}= \eta(3005)$	$\eta_C(2980)$, Γ=17.3 Mev
$q_N(313)\overline{d^1_S(1933)}$=K(2076)	$K^*_4(2045)$, Γ = 198 $\pm$ 30 Mev
$d^1_S(493)\overline{d^1_S(1933)}=\eta(2127)$	$f_2(2150)$, Γ= 167$\pm$30 Me

D4. The d^1_b(4913)-quark (Table 7)

The d^1_b(4913)-quark has Q = -$\frac{1}{3}$, $\overrightarrow{n}$ =(3, 3, 0). Table 44 shows its evidence:

Table 44. The Evidence of the d^1_S(4913)-quark

$d^1_b(4913)u(313)d(313)=\Lambda_b(5539)$	$\Lambda_b(5624)$, $\tau=(1.229\pm.080)\times10^{-12}$s;
$d^1_b(4913)\overline{d^1_b(4913)}=\Upsilon(1S)(9389)$	$\Upsilon(1S)(9460)$, Γ=(53.0 $\pm$1.5) kev
$q_N(313)\overline{d^1_b(4913)}$= B(5378)	B(5279), $\tau=(1.671\pm.018)\times10^{-12}$s;
$d^1_b(493)\overline{d^1_b(4913)}= B_S(5344)$	$B_S(5370)$, Γ= (1.461 $\pm$ 0.057) $\times10^{-12}$s]

In Table 41-Table 44, we can see that the four brother quarks [d(313), d^1_S(493), d^1_S(1933) and d^1_b(4913)] really do exist.

Please pay attention to $\overrightarrow{n}$ = (n_1, n_2, n_3) values of the brothers of the three families: for the first one, n =(0, 0, 2), n =(0, 0, 4), n =(0, 0, 6); for the second one, $\overrightarrow{n}$= (0, 0, 0), $\overrightarrow{n}$= (1, 1, 0), $\overrightarrow{n}$= (2, 2, 0), $\overrightarrow{n}$= (3, 3, 0); for the third one $\overrightarrow{n}$ =(0, 0, 0), $\overrightarrow{n}$ =(0, 0, -2), $\overrightarrow{n}$ =(0, 0, -4). These values are very interesting. They might be hitting a law or a solution of an equation. Since each $\overrightarrow{n}$ represents a discovered quark, they might be implicating a physical law or a fundamental physical equation.

As with Table 32-Table 44, we can show that most of the deduced quarks in Table 11 have already been discovered by experiments.

E Experiments Have Already Discovered Most of the Deduced Quarks in Table 11

Using the phenomenological formulae, this paper deduces an excited quark spectrum, shown in Table 10 and Table 11. Experiments have already discovered most of these quarks inside baryons and mesons. The q_N(m)-quarks are inside the N-baryons (Table 18), the light unflavored mesons with I = 0 (Table 27) and the strange K(M)-mesons (Table 31). The q_Δ(m)-quarks are inside the Δ-baryons (Table 18) and the light unflavored mesons with I = 1 (Table 28). The d_S(m)-quarks are inside the Λ-baryons (Table 17), the light unflavored mesons with I = 0 (Table 27) and I = 1 (Table 28), the strange K-mesons (Table 31) and the heavy unflavored mesons (Table 25). The q_Σ(m)-quarks are inside the Σ-baryons (Table 17) and the light unflavored mesons with I = 1 (Table 28). The u_C(m)-quarks are inside the Λ_C^+-baryons (Table 14 and Table 20), the D-mesons (Table 30) and the intermediate mass ψ-mesons (Table 26). The d_b(m)-quarks are inside the Λ_b^0-baryon (Table 20), the B(M)-mesons (Table 29) and the Υ-meson (Table 25). The q_{Ξ_C}(m)-quarks are inside the Ξ_C-baryons (Table 21). The Ω_C(2133)-quark is inside the Ω_C(2907)-baryon (Table B7, Table 14 and Table 20). The u_C(6023)-quark (Table 4) is inside the meson Υ(10355) (Table 25).

These experimental results are enough to support the deduced quark spectrum and the phenomenological formulae.

It is very important to pay attention to the Υ(3S)-meson (mass m = 10.3052 $\pm$ 0.00004 Gev, full width Γ = 26.3 $\pm$ 3.5 kev). Υ(3S) has more than three times larger mass than J/ψ(1S) (m = 3096.87 $\pm$ 0.04 Mev) and more than three times longer lifetime than J/ψ(1S) (full width Γ = 87 $\pm$ 5kev). It is well known that the discovery of J/ψ(1S) is also the discovery of charmed quark c (u_c(1753)). Similarly the discovery of Υ(3S) will be the discovery of a very important new quark—the u_C(6073)-quark.

VII Predictions

A New Quarks, Baryons and Mesons

This paper predicts many quarks, baryons and mesons with high rest masses. Since these particles have high rest masses, they are difficult to discover. The following quarks, baryons and mesons in Table 45 have a possibility of being discovered in the not too distant future:

Table 45. The Important Predictive Quarks, Baryons and Mesons

	n_1,n_2,n_3	Quark	Baryon	$q(m)\overline{q(m)}$	$q_N(313)\overline{q}$	$d_S(493)\overline{q}$
$E_P=\frac{27}{4}$	(2, 2, 2)	$d_S^0(2743)$	$\Lambda(3369)$	$\eta(4196)$	K(2786)	$\eta(3047)$
$E_H= 9$	(0, 0, 4)	$d_S^{-1}(3753)$	$\Lambda(4379)$	$\eta(5472)$	K(3896)	$\eta(4156)^*$
$E_H= 25$	(0, 0, 6)	$d_S^{-1}(9613)$	$\Lambda(10239)$	$\eta(18133)$	K(9781)	$\eta(10056)^*$
$E_\Gamma=16$	$(0, 0, \overline{4})$	$u_C^1(6073)$	$\Lambda_C(6599)$	$\psi(10509)^*$	D(6231)	$D_S(6312)$
$E_N=\frac{49}{2}$	(4, 4, 0)	$d_b^1(9333)$	$\Lambda_b(9959)$	$\Upsilon(17868)$	B(9503)	$B_S(9579)$

* Some mesons have been discovered: $u_C(6073)\overline{u_C(6073)}=\psi(10509)^*[\Upsilon(10355)]$; $d_S(9613)\overline{d_S(493)}=\eta(10056)^*[\Upsilon(10023)]$ and $d_S^{-1}(3753)\overline{d_S(493)}=\eta(4156)^*[\psi(4160)]$.

The $\Lambda_C(6599)$-baryon, the D(6231)-meson, the K(2786)-meson and the $\Lambda_b(9959)$-baryon have a better chance of being discovered. In order to confirm the $u_C(6073)$-quark, we need to find the $\Lambda_C(6599)$-baryon first.

B The "fine structure" of Particle Physics

The large full widths of some mesons and baryons might indicate a "fine structure" of particle physics. There are several mesons (baryons) that have the same intrinsic quantum numbers (I, S, C, b and Q), angular momentums and parities but different rest masses. They are different mesons (baryons). Since there are only five quarks in current Quark Model, physicists think that there is only one meson (baryon) with large full width. For example, $f_0(600)$ might be $q_N(313)\overline{q_N(583)}$ + $q_N(583)\overline{q_N(583)}$ + $q_\Sigma(583)\overline{q_\Sigma(583)}$ + $q_\Delta(673)\overline{d_\Delta(673)}$ + $q_\Xi(673)\overline{d_\Xi(673)}$, (see Table C5), shown in Table 46

.

Table 46. The "Fine Structure" of $f_0(600)$ [12]

$q_N(313)\overline{q_N(583)}$	-320	$\eta(576)$	
$q_N(583)\overline{q_N(583)}$	-528	$\eta(638)$	$f_0(600)$
$q_\Sigma(583)\overline{q_\Sigma(583)}$	-676	$\eta(676)$	Γ= 600-1000
$q_\Delta(673)\overline{d_\Delta(673)}$	-938	$\eta(408)$	
$q_\Xi(673)\overline{d_\Xi(673)}$	-538	η (808)	

Thus, we propose that experimental physicists separate both the mesons and the baryons with large widths using experiments. First, we shall separate the $f_0(600)$-meson (with Γ= 600-1000 Mev). The experimental investigation of the predicted "fine structure" for both mesons and baryons with large widths provides a crucial test for our phenomenological formulae.

VIII Discussion

1. These phenomenological formulae not only decrease the number of elementary quarks, but also increase the number of excited quarks that compose new baryons and mesons. The decrease in the number of elementary quarks will decrease the number of parameters and may simplify the calculations. The increase in the number of excited quarks provides the explanation for many newly discovered high-mass baryons and mesons.

2. Since individual free quarks cannot be found, we cannot directly measure the binding energies of the baryons and mesons. Although we cannot measure these binding energies, they do exist. The fact that physicists have not found individual free quarks shows that the binding energies of the baryons and the mesons are huge. The binding energies (-3Δ) (54) of the baryons [or mesons (-2Δ) (59)] are always canceled by the corresponding parts (3Δ) of three quarks [or two quarks (2Δ)] (see Table 11). Thus, we

can omit the binding energies [baryons (-3Δ) and mesons (-2Δ)] and the corresponding part (Δ) (see Table 11) of the rest masses of the quarks when we calculate the rest masses of the baryons and the mesons. The effect looks as if there is not binding energy in baryons and mesons.

3. The phenomenological formulae only apply to strong interaction static properties of quarks, baryons and mesons. They do not apply to electromagnetic and weak interactions. They cannot work for strong interaction scattering processes.

4. These formulae do not break any principles of the Quark Model, such as SU(n) symmetries, the sum laws, the"qqq" baryon model and the "q$\overline{q}$" meson model. In fact, they preserve and improve these principles. They show that all quarks are the excited state of the same elementary quark ϵ to provide a physical foundation of SU(n) symmetries. They deduce many new quarks to substantiate the"qqq" baryon model and the "q$\overline{\text{q}}$" meson model. They deduce the rest masses and the intrinsic quantum numbers of quarks, baryons and mesons to consolidate the Quark Model. They infer the large binding energies to strengthen the foundation of the confinement of the quarks.

IX Conclusions

1. There is only one unflavored elementary quark family ϵ with three colors that have two isospin states (ϵ_u with $I_Z = \frac{1}{2}$ and $Q = +\frac{2}{3}$, ϵ_d with $I_Z = \frac{-1}{2}$ and $Q = -\frac{1}{3}$) for each color. Thus there are six Fermi ($s = \frac{1}{2}$) elementary quarks with S = C = b = 0 in the vacuum.

2. When an elementary quark (ϵ) with one color (red, yellow or blue) and a charge Q (= $\frac{2}{3}$ or -$\frac{1}{3}$) is excited from the vacuum, its color and electric charge will remain unchanged. At the same time, it may go into an energy band of (2) to get the rest mass and intrinsic quantum numbers (I, S, C, and b) and become an excited quark.

3. Using the phenomenological formulae, we deduced an excited quark spectrum of

the ϵ-quarks (see Table 10 and Table 11) from the elementary ϵ-quarks. Experiments have already provided evidence for almost all of the deduced quarks inside the baryons (see Table 16-Table 21) and mesons (see Table 25-Table 31). All quarks inside the baryons [11] and the mesons [12] are the excited states of the elementary ϵ-quarks.

4. The five quarks of the current Quark Model correspond to the five deduced ground quarks [u$\leftrightarrow$u(313), d$\leftrightarrow$d(313), s$\leftrightarrow$$d_s$(493), c$\leftrightarrow$$u_c$(1653) and b$\leftrightarrow$$d_b$(4913)] (see Table 11). The current Quark Model uses only the five independent "elementary" quarks to explain the baryons and mesons, there is no excited quark spectrum and no phenomenological formulae in the model. Thus, the current Quark Model is the five ground quark approximation of the new Quark Model with the phenomenological formulae.

5. With the phenomenological formulae, we deduce the rest masses and the intrinsic quantum numbers (I, S, C, b and Q) of the quarks, baryons and mesons. These deduced intrinsic quantum numbers match exactly with the experimental results. The deduced rest masses are consistent with experimental results at a 98% level of confidence. Without the phenomenological formulae, the current Quark Model cannot deduce the rest masses of the quarks, baryons and mesons. With the formulae, the Quark Model is much more powerful.

6. This paper has deduced the intrinsic quantum numbers of the quarks using the following formulae: a). the strange numbers are from S = R - 4 (6); b). the isospins are from deg = 2I + 1 (8); c). the charmed numbers are from (20); d). the bottom numbers are from (21); e). the electric charges are from the elementary quarks (24) and (25). From the deduced intrinsic quantum numbers of the quarks, using the sum laws, this paper has deduced the intrinsic quantum numbers of the baryons and mesons.

7. Using the phenomenological mass formula (37), this paper has deduced the rest masses of the quarks (see Table 11). From these deduced rest masses of the quarks, using the sum laws and the phenomenological binding energy formula (55) and (59), this paper has deduced the baryon mass spectrum (see Table 16-Table 21) and the

meson mass spectrum (see Table 25-Table 31) respectively. The deduced rest masses of the baryons and mesons are consistent with the experimental results (with a 98% level of confidence).

8. The SU(3) (based on u(313), d(313) and $d_S(493)$), SU(4), SU(5), ... SU(n), ... are natural extensions of the SU(2) (based on ϵ_u and ϵ_d). These phenomenological formulae provide a physical foundation for SU(3)... ,SU(n), ...

9. For baryons, the binding energy is roughly a constant (55). For mesons, the binding energy is roughly a constant (59) too. These binding energies are the phenomenological approximations of the color's strong interaction energies. Although we do not know their exact values, we believe that the binding energies of the baryons and mesons are very large. These large binding energies provide a possible foundation for the confinement of the quarks inside the hadrons.

10. The experimental investigation of the "fine structure" for both the mesons and baryons with large widths (for example $f_0(600)$ with Γ = 600-1000 [12]), provides a crucial test for our phenomenological formulae.

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Appendix A. The Regular Rhombic Dodecahedron and $\vec{n}$ Values

There are many symmetry points (Γ, H, P, N, M, ...) and symmetry axes (Δ, Λ, Σ, D, F, G, ...) in the regular rhombic dodecahedron (see Fig. 1).

We give a definition of the equivalent symmetry points: taking $\overrightarrow{n}$ (n_1, n_2, n_3) = 0 in (2), if for different points $\overrightarrow{k}$ = (ξ, η, ς), we can get the same E($\vec{k}$,$\vec{n}$) value. We call these points the "equivalent" points. Table A1 shows the equivalent points of the regular rhombic dodecahedron:

Table A1. The Equivalent Points

Sym. Point (ξ, η, ς)	The Equivalent Points (ξ, η, ς)
Γ = (0, 0, 0)	(0, 0, 0)
P = $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$	$(\pm\frac{1}{2}, \pm\frac{1}{2}, \pm\frac{1}{2})$
H = (0, 0, 1)	$(0, 0, \pm 1)$, $(0, \pm 1, 0)$, $(\pm 1, 0, 0)$
N = $(\frac{1}{2}, \frac{1}{2}, 0)$	$(\pm\frac{1}{2}, \pm\frac{1}{2}, 0)$, $(\pm\frac{1}{2}, 0, \pm\frac{1}{2})$, $(0, \pm\frac{1}{2}, \pm\frac{1}{2})$

Similarly, we can give the definition of the equivalent symmetry axes. Table A2 shows the equivalent axes inside the regular rhombic dodecahedron:

Table A2. The Equivalent Symmetry Axes

Symmetry Axis	Equivalent Symmetry Axes
Γ(000)-H(001)	Γ-$(0, \pm 1, 0)$, Γ-M(± 1,0,0), Γ-H'(0,0,-1)
Γ(000)-P($\frac{1}{2}\frac{1}{2}\frac{1}{2}$)	Γ-P'$(\pm\frac{1}{2}, \pm\frac{1}{2}, \pm\frac{1}{2})$
Γ(000)-N($\frac{1}{2}\frac{1}{2}0$)	Γ-N'$(\pm\frac{1}{2}, \pm\frac{1}{2}, 0)$, Γ-N'$(\pm\frac{1}{2}, 0, \pm\frac{1}{2})$, Γ-N'$(0, \pm\frac{1}{2}, \pm\frac{1}{2})$

In order to distinguish the axes inside the regular rhombic dodecahedron from the axes on its surfaces, we define an in-out number Θ (for Θ = 0, the axis is inside; for Θ=1, the axis is on the surface):

$$\Theta = \overrightarrow{k_{Start}} \bullet \overrightarrow{k_{End}} = 2(\xi_{Start} \times \xi_{End} + \eta_{Start} \times \eta_{End} + \varsigma_{Start} \times \varsigma_{End}). \tag{65}$$

For the axes inside it, Θ =0; for the axes on its surfaces, $\theta = 1$. Table A3 shows the in-out number Θ, the symmetry rotary fold R, the strange number S, the symmetry operation P of the symmetry axes and the first (second) division K_{fir}= 0 (K_{Sec}=1):

Table A3. The Θ, R, S and Symmetry Operation(p) Values

Axis	Start Point	End Point	Θ	R	S	P	K_{first}	K_{Secend}
$\Delta(\Gamma$-H)	(0, 0, 0)	(0, 0, 1)	0	4	0	8	0	/
$\Lambda(\Gamma$-P)	(0, 0, 0)	$(\frac{1}{2},\frac{1}{2},\frac{1}{2})$	0	3	-1	6	0	/
$\Sigma(\Gamma$-N)	(0, 0, 0)	$(\frac{1}{2},\frac{1}{2},0)$	0	2	-2	4	0	/
D(P-N)	$(\frac{1}{2},\frac{1}{2},\frac{1}{2})$	$(\frac{1}{2},\frac{1}{2},0)$	1	2	0	4	0	/
F(P-H)	$(\frac{1}{2},\frac{1}{2},\frac{1}{2})$	(0, 0, 1)	1	3	-1	6	0	$1^{\#}$
G(M-N)	(1, 0, 0)	$(\frac{1}{2},\frac{1}{2},0)$	1	2	-2	4	0	$1^{\#}$

$^{\#}$K=1 corresponds to the second divisions in sixfold bands.

From (2) we can also give a definition of the equivalent $\overrightarrow{n}$: for $\xi = \eta = \varsigma = 0$, all $\overrightarrow{n}$ values that yield a same E($\overrightarrow{n}$,0) value are equivalent n-values. We show the low level equivalent $\overrightarrow{n}$-values that satisfy conditions (3) in the following list (66) (note $\overline{n_i}$ = - n_i):

E($\overrightarrow{n}$,0) = 0 :	(0, 0, 0)
E($\overrightarrow{n}$,0) = 2 :	(101, $\bar{1}01$, 011, $0\bar{1}1$, 110, $1\bar{1}0$, $\bar{1}10$, $\bar{1}\bar{1}0$, $10\bar{1}$, $\bar{1}0\bar{1}$, $01\bar{1}$, $0\bar{1}\bar{1}$)
E($\overrightarrow{n}$,0) = 4 :	(002, 200, 200, $\bar{2}00$, $0\bar{2}0$, $00\bar{2}$)
E($\overrightarrow{n}$,0) = 6:	112, 211, 121, $\bar{1}21$,$\bar{1}12$, $2\bar{1}1$, $1\bar{1}2$, $21\bar{1}$,$12\bar{1}$,$\bar{2}11$, $1\bar{2}1$, $11\bar{2}$, $\bar{1}\bar{1}2$, $\bar{1}2\bar{1}$, $2\bar{1}\bar{1}$, $\bar{2}\bar{1}1$, $\bar{1}\bar{2}1$, $1\bar{1}\bar{2}$, $1\bar{2}\bar{1}$,$\bar{1}\bar{1}\bar{2}$,$\bar{2}\bar{1}\bar{1}$, $\overline{211}$, $\overline{121}$, $\overline{112}$,
E($\overrightarrow{n}$,0) = 8:	(220, $2\bar{2}0$, $\bar{2}20$, $\bar{2}\bar{2}0$, 202, $20\bar{2}$, $\bar{2}02$, $\bar{2}0\bar{2}$, 022, $02\bar{2}$, $0\bar{2}2$, $0\bar{2}\bar{2}$)

(66)

Table A4. The Energy Bands with ΔS = 0

Axis	Δ	Σ	Λ	Λ	D	F	G
R	4	2	3	3	2	3	2
S_{Ax}	0	-2	-1	-1	0	-1	-2
deg	4	2	3	1	2	1	2
ΔS	0	0	0	0	0	0	0
q_{Name}	q_{Δ}	q_{Ξ}	q_{Σ}	d_S	q_N	d_S	q_{Ξ}

Table A5. The Energy Bands with the Fluctuation $\Delta S \neq 0$

Axis	Δ	Δ	Σ	Σ	Σ	D	D	F	F	F	G	G
R	4	4	2	2	2	2	2	3	3	3	2	2
S_{Ax}	0	0	-2	-2	-2	0	0	-1	-1	-1	-2	-2
deg	1	1	1	1	1	1	1	2	2	2	1	1
ΔS	1	-1	1	1	-1	1	-1	1	1	-1	1	-1
S	0	-1	-1	0	-3	0	-1	0	-1	-2	-2	-3
C	1	0	0	0	0	1	0	0	1	0	1	0
b	0	0	0	-1	0	0	0	0	0	0	0	0
quark	u_C	d_S	d_S^*	$d_b^\#$	d_Ω	u_C	d_S	q_N	q_{Ξ_C}	q_Ξ	d_{Ω_C}	d_Ω

*If $\Delta E = 0$, the quark is d_S; $^\#$If $\Delta E \neq 0$, the quark is d_b.

A6 The Possible Maximum Isospin of baryons at symmetry points

Symmetry Axis	Δ	Δ	Σ	Σ	Λ	Λ	D	D	F	F	G	G
Symmetry Point	Γ	H	Γ	N	Γ	P	P	N	P	H	M	N
Symmetry rotatory R	4	4	2	2	3	3	2	2	3	3	2	2
Maximum Equ-n Value	8	8	4	4	6	6	2	2	2	2	2	2
Maximum I of Quarks	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
Possibility Large I_{Baryon}	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	1	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{1}{2}$

Appendix B. Energy Bands and Corresponding Quarks

From Table 2, Table 3 and Table 4, omitting Δ part mass of quarks, we have the quarks (q_Δ^0, d_S^{-1} and u_C^1) shown in Table B1 (note $\overline{n_i}$ = - n_i):

Table B1. The Energy Bands and the Quarks of the Δ-Axis (S = 0)

E_{Start}	$E(\vec{k},\vec{n})$	d	ΔS	J	($n_1n_2n_3$, ...); J	S	C	ΔE	q(m $_{(\mathrm{Mev})}$)
E_Γ=0	313	1	0	0	(000): $J_\Gamma = 0$;	0	0	0	u(313)
E_H=1	673	4	0	0	$(101,\bar{1}01,011,0\bar{1}1)$	0	0	0	$q_\Delta(673)$
E_H=1	673	1	-1	1	(002); $J_{S,H} = 1$	-1	0	100	$d_S^{-1}(773)$
E_Γ=2	1033	4	0	0	$(110,1\bar{1}0,\bar{1}10,\bar{1}\bar{1}0)$	0	0	0	$q_\Delta(1033)$
E_Γ=2	1033	4	0	0	$(10\bar{1},\bar{1}0\bar{1},01\bar{1},0\bar{1}\bar{1})$	0	0	0	$q_\Delta(1033)$
E_H=3	1393	4	0	0	$(112,1\bar{1}2,\bar{1}12,\bar{1}\bar{1}2)$	0	0	0	$q_\Delta(1393)$
E_Γ=4	1753	4	0	0	$(200,\bar{2}00,020,0\bar{2}0)$	0	0	0	$q_\Delta(1753)$
E_Γ=4	1753	1	1	1	$(00\bar{2})$; $J_{C,\Gamma} = 1$	0	1	0	$u_C^1(1753)$
E_H=5	2113	4 4	0	0	$(121,1\bar{2}1,\bar{1}21,\bar{1}\bar{2}1)$ $(211,2\bar{1}1,\bar{2}11,\bar{2}\bar{1}1)$	0	0	0	$q_\Delta(2113)$ $q_\Delta(2113)$
E_H=5	2113	4	0	0	$(202,\bar{2}02,022,0\bar{2}2)$	0	0	0	$q_\Delta(2113)$
E_H=5	2113	4	0	0	$(013,0\bar{1}3,103,\bar{1}03)$	0	0	0	$q_\Delta(2113)$
E_Γ=6	2473	4 4	0	0	$(12\bar{1},1\bar{2}\bar{1},\bar{1}2\bar{1},\bar{1}\bar{2}\bar{1})$ $(21\bar{1},2\bar{1}\bar{1},\bar{2}1\bar{1},\bar{2}\bar{1}\bar{1})$	0	0	0	$q_\Delta(2473)$ $q_\Delta(2473)$
E_Γ=6	2473	4	0	0	$(11\bar{2},1\bar{1}\bar{2},\bar{1}1\bar{2},\bar{1}\bar{1}\bar{2})$	0	0	0	$q_\Delta(2473)$
E_H=9	3553	1	-1	2	(004); $J_{S,H} = 2$	-1	0	200	$d_S^{-1}(3753)$
E_Γ=16	6073	1	1	2	$(00\bar{4})$; $J_{C,\Gamma} = 2$	0	1	0	$u_C^1(6073)$
E_H=25	9313	1	-1	3	(006); $J_{S,H} = 3$	-1	0	300	$d_S^{-1}(9613)$
E_Γ=36	13273	1	1	3	$(00\bar{6})$; $J_{C,\Gamma} = 3$	0	1	0	$u_C^1(13273)$
E_H=49	17953	1	-1	4	(008); $J_{S,H} = 4$	-1	0	400	$d_S^{-1}(18353)$
...	...	...	...	...	...	...	...	...	....

$E_{\mathrm{Start}} = [(n_1\text{-}\xi)^2+(n_2\text{-}\eta)^2+(n_3\text{-}\zeta)^2]$, $E(\vec{k},\vec{n})$ is the starting and minimum energy of the energy band; $\overline{n_i}$ = -n_i; $\overline{n_1n_2n_3}$ = -n_1, -n_2, -n_3 [see Eq. (66)].

From Table 2, Table 5, Table 6 and Table 7, omitting Δ part mass of quarks, we have the quarks (q^0_Ξ, d^1_S , d^1_b and d^{-1}_Ω) shown in Table B2 (note $\overline{n_i}$ = - n_i):

Table B2. The Energy Bands and the Quarks of the Σ-Axis (S = -2)

E_{Start}	$E(\vec{k},\vec{n})$	d	ΔS	J	$(n_1n_2n_3)$; J	S	b	ΔE	q(m $_{(Mev)}$)
E_Γ= 0	313	1	-2	0	(000); J_Γ=0	0	0	0	d(313)
E_N=1/2	493	2	1	1	(110); J_N=1;	-1	0	0	d^1_S(493)
E_N=3/2	853	2 2	0	0	$(101,10\bar{1})$ $(011,01\bar{1})$	-2	0	0	q^0_Ξ(853) q^0_Ξ(853)
E_Γ= 2	1033	2	0	0	$(1\bar{1}0,\bar{1}10)$	-2	0	0	q^0_Ξ(1033)
E_Γ= 2	1033	2 2	0	0	$(\bar{1}01,\bar{1}0\bar{1})$ $(0\bar{1}1,0\bar{1}\bar{1})$	-2	0	0	q^0_Ξ(1033) q^0_Ξ(1033)
E_Γ= 2	1033	2	-1	1	$(\bar{1}\bar{1}0)$; J_Γ=1	-3	0	0	d^{-1}_Ω(1033)
E_N=5/2	1213	2	0	0	(200,020)	-2	0	0	q^0_Ξ(1213)
E_N=7/2	1573	2 2	0 0	0	$(121,12\bar{1})$ $(211,21\bar{1})$	-2	0	0	q^0_Ξ(1573) q^0_Ξ(1573)
E_Γ= 4	1753	2	0	0	$(002,00\bar{2})$	-2	0	0	q^0_Ξ(1753)
E_Γ= 4	1753	2	0	0	$(\bar{2}00,0\bar{2}0)$	-2	0	0	q^0_Ξ(1753)
E_N=9/2	1933	2	0	0	$(112,11\bar{2})$	-2	0	0	q^0_Ξ(1933)
E_N=9/2	1933	1	1	2	(220); J_N=2	-1	0	0	d^1_S(1933)
$E_N=\frac{11}{2}$	2293	1	0	0	$(\bar{1}21,\bar{1}2\bar{1},$ $2\bar{1}1,2\bar{1}\bar{1})$	-2	0	0	q^0_Ξ(2293) q^0_Ξ(2293)
E_Γ= 8	3193	1	-1	2	$(\bar{2}\bar{2}0)$; J_Γ=2;	-3	0	0	d^{-1}_Ω(3193)
E_N=25/2	4813	1	1	3	(330); J_N=3;	0	-1	100	d^1_b(4913)
E_Γ= 18	6793	1	-1	3	$(\bar{3}\bar{3}0)$; J_Γ=3;	-3	0	-300	d^{-1}_Ω(6493)
E_N=49/2	9133	1	1	4	(440); J_N=4;	0	-1	200	d^1_b(9333)
E_Γ= 32	11833	1	-1	4	$(\bar{4}\bar{4}0)$; J_Γ=4;	-3	0	-600	d^{-1}_Ω(11233)
E_N=81/2	14893	1	1	5	(550); J_N=5;	0	-1	300	d^1_b(15193)
...	...	...	...	...	...	...	...	...	....

$E_{Start} = [(n_1-\xi)^2+(n_2-\eta)^2+(n_3-\zeta)^2]$, $E(\vec{k},\vec{n})$ is the starting and minimum energy of the energy band; $\overline{n_i}$ = -n; $\overline{n_1n_2n_3}$ = $-n_1$, $-n_2$, $-n_3$ [see Eq. (66)].

From Table 2, Table 8 and Table 9, omitting Δ part mass of quarks, we have the

quarks (q^0_Σ and d^0_S) shown in Table B3 (note $\overline{n_i}$ = - n_i);

Table B3. The Energy Bands and the Quarks of the Λ-Axis (S = -1)

E_{Start}	$E(\vec{k},\vec{n})$	d	ΔS	J	$(n_1 n_2 n_3, ...)$	S	$q(m_{(Mev)})$
E_Γ=0	313	1	-1	0	(000)	0	d(313)
E_P=3/4	583	3	0	0	(101,011,110)	-1	q^0_Σ(583)
E_Γ= 2	1033	3 3	0	0	$(1\bar{1}0,\bar{1}10,01\bar{1})$ $(0\bar{1}1,10\bar{1},\bar{1}01)$	-1	q^0_Σ(1033) q^0_Σ(1033)
E_Γ= 2	1033	3	0	0	$(\bar{1}0\bar{1},0\bar{1}\bar{1},\bar{1}\bar{1}0)$	-1	q^0_Σ(1033)
E_P=11/4	1303	3	0	0	(020,002,200)	-1	q^0_Σ(1303)
E_P=11/4	1303	3	0	0	(121,211,112)	-1	q^0_Σ(1303)
E_Γ= 4	1753	3	0	0	$(0\bar{2}0,\bar{2}00,00\bar{2})$	-1	q^0_Σ(1753)
E_P=19/4	2023	3 3	0	0	$(1\bar{1}2,\bar{1}12,21\bar{1})$ $(2\bar{1}1,12\bar{1},\bar{1}21)$	-1	q^0_Σ(2023) q^0_Σ(2023)
E_P=19/4	2023	3	0	0	(202,022,220)	-1	q^0_Σ(2023)
E_Γ= 6	2473	3 3	0	0	$(\bar{2}11,2\bar{1}\bar{1},\bar{1}\bar{1}2)$ $(11\bar{2},\bar{1}2\bar{1},1\bar{2}1)$	-1	q^0_Σ(2473) q^0_Σ(2473)
E_Γ= 6	2473	3 3	0	0	$(\bar{1}\bar{2}1,1\bar{2}\bar{1},\bar{1}1\bar{2})$ $(1\bar{1}\bar{2},\bar{2}1\bar{1},\bar{2}\bar{1}1)$	-1	q^0_Σ(2473) q^0_Σ(2473)
E_Γ= 6	2473	3	0	0	$(\bar{1}\bar{2}\bar{1},\bar{1}\bar{1}\bar{2},\bar{2}\bar{1}\bar{1})$	-1	q^0_Σ(2473)
E_P=27/4	2743	3 3	0	0	(013,031,310) (130,301,103)	-1	q^0_Σ(2743) q^0_Σ(2743)
	2743	1	0	0	(222)	-1	d^0_S(2743)

For the D-, F- and G-axes on the surfaces of the regular rhombic dodecahedron (see Fig. 1), the energy bands with the same energy might not have all equivalent $\vec{n}$ values. Using (10), (11) and (8) [the first division, K = 0], we can get isospin and other intrinsic quantum numbers.

For sixfold energy bands of the F-axis and the G-axis, we need **a second division, K = 1**. Using (22) and (23), we can obtain the q_{Ξ_C}-quark and the q_{Ω_C}-quark, shown in Table B6 and Table B7.

There are three energy bands ($\vec{n}$ = (000), $\vec{n}$ = (100) and $\vec{n}$ = (200)) that have

already been recognized on the three axes Γ-H, Γ-P and Γ-N. The bands on the surfaces of the regular rhombic dodecahedron are the same quarks as those inside:

Table B4. The Special Energy bands

$\overrightarrow{n}$	Bands (Inside dodecahedron)	Bands (on Surface)	Quark
(000)	$E_\Gamma(0) \rightarrow E_N(\frac{1}{2})$ $E_\Gamma(0) \rightarrow E_p(\frac{3}{4})$ $E_\Gamma(0) \rightarrow E_H(1)$	$E_N(\frac{1}{2}) \rightarrow E_p(\frac{3}{4})$ $E_p(\frac{3}{4}) \rightarrow E_H(1)$ $E_N(\frac{1}{2}) \rightarrow E_M(1)$	$q_N(313)$
(110) $(1\bar{1}0)$	$E_N(\frac{1}{2}) \rightarrow E_\Gamma(2)$	$E_N(\frac{1}{2}) \rightarrow E_p(\frac{3}{4})$ $E_p(\frac{3}{4}) \rightarrow E_H(3)$ $E_N(\frac{1}{2}) \rightarrow E_M(1)$ $E_M(1) \leftarrow E_N(\frac{5}{2})$	$d_S(493)$
(200)	$E_H(1) \rightarrow E_\Gamma(2)$	$E_M(1) \rightarrow E_N(\frac{5}{2})$,	$d_S(773)$
(002)		$E_H(1) \rightarrow E_p(\frac{11}{4}) \rightarrow E_N(\frac{9}{2})$	$d_S(773)$

For each symmetry axis, from (30), we can get simple binding energy formulae:

For the D-axis, $\Theta = 1$, CI = CS = K = b = $S_{Ax} = 0$, from(30):

twofold, $\Delta E = 100(2CJ_C + SJ_S\Delta S)$ $\quad J_C = 1, 2, 3, \ldots, J_{S,2} = 2, 3, \ldots$;
$\Delta E = 0$, $J_{S,2} = 1$; (67)

fourfold, $C = \Delta S = 0$, $\Delta E = 0$.

For the F-Axis, $\Theta = 1$, $b = 0$, $S_{Ax} = -1$, from(30), we have:

threefold, $K = C = (1+S_{Ax}) = 0$, $\Delta E = 0$; (68)

sixfold, $\quad K=1$, $\Delta E = 100[C(2J_C\text{-}I)\text{-}CS+S]$, $\quad J_C = 1, 2, 3, \ldots$;

For the G-Axis, $\Theta = 1$, $b = 0$, $S_A = -2$. From (30), we have:

twofold $C = K = 0$, $\Delta E = (-S)(J_{S,2}-2)\Delta S$ $\quad J_{S,2} = 4, 5\ 6, \ldots$; $\Delta E = 0$ $\quad J_{S,2}, \leq 3$;

fourfold $C = K = \Delta S = 0$, $\Delta E = 0$;

$$\text{sixfold } K = 1, \left\{ \begin{array}{l} \Delta E = 100\{C(2J_C\text{-}S)+S\ \text{-}S(J_{S,6}-2)\Delta S\}\ J_C=1, 2, \ldots; \\ J_{S,6} = 4, 5, \ldots\ .\ \ \Delta E = 0,\ J_{S,6} < 4. \end{array} \right\} \qquad (69)$$

For the D-axis, from Table 2, (2), (66), (14), (20), (67), (37) and Table B4, omitting Δ part energy of quarks, we have:

Table B5. The Energy Bands and the Quarks of the D-Axis (S = 0)

E_{Start}	E	$(n_1n_2n_3)$,...	ΔS	J	S	C	ΔE	q(m $_{(Mev)}$)
$E_N = \frac{1}{2}$	493	(000) (110)	0 -1	0 $J_{S,2}$=1	0 -1	0 0	0	u(313) $d_S^{-1}(493)$
$E_P = \frac{3}{4}$	583	(101, 011)	0	0	0	0	0	$q_N^0(583)$
$E_N = \frac{3}{2}$	853	$(10\overline{1}, 01\overline{1})$	0	0	0	0	0	$q_N^0(853)$
$E_N = \frac{5}{2}$	1213	$(1\overline{1}0, \overline{1}10)$ (020, 200)	0 0	0 0	0 0	0 0	0	$q_N^0(1213)$ $q_N^0(1213)$
$E_P = \frac{11}{4}$	1303	$(\overline{1}01, 0\overline{1}1,$ (211, 121)	0 0	0 0	0 0	0 0	0	$q_N^0(1303)$ $q_N^0(1303)$
$E_P = \frac{11}{4}$	1303	(002) (112)	-1 -1	$J_{S,2}$=2 $J_{S,2}$=3	-1 -1	0 0	200 300	$d_S^{-1}(1503)$ $d_S^{-1}(1603)$
$E_N = \frac{7}{2}$	1573	$(12\overline{1}, 21\overline{1})$ $(\overline{1}0\overline{1}, 0\overline{1}\overline{1})$	0 0	0 0	0 0	0 0	0	$q_N^0(1573)$ $q_N^0(1573)$
$E_N = \frac{9}{2}$	1933	(220, $\overline{1}\overline{1}0$)	-1 1	$J_{S,2}$=4 J_C= 1	-1 0	0 1	400 200	$d_S^{-1}(2333)$ $u_C(2133)$
$E_N = \frac{9}{2}$	1933	$(11\overline{2})$ $(00\overline{2})$	1 1	J_C=2 J_C=3	0 0	1 1	400 600	$u_C(2333)$ $u_C(2533)$
$E_p = \frac{19}{4}$	2023	$(\overline{1}21, 2\overline{1}1)$	0	0	0	0	0	$q_N^0(2023)$
$E_P = \frac{19}{4}$	2023	$(\overline{1}12, 1\overline{1}2)$ (202,022)	0 0	0 0	0 0	0 0	0	$q_N^0(2023)$ $q_N^0(2023)$
$E_N = \frac{11}{2}$	2293	$(2\overline{1}\overline{1}, \overline{1}2\overline{1})$	0	0	0	0	0	$q_N^0(2293)$
$E_N = \frac{13}{2}$	2653	(310, 130) $(\overline{2}00, 0\overline{2}0)$	0 0	0 0	0 0	0 0	0	$q_N^0(2653)$ $q_N^0(2653)$
$E_P = \frac{27}{4}$	2743	(222)	1	J_C=4	0	1	800	$u_C(3543)$

$E_{Start} = [(n_1-\xi)^2+(n_2-\eta)^2+(n_3-\zeta)^2]$, E is the starting and minimum energy of the energy band; $\overline{n_i}$ = -n; $\overline{n_1n_2n_3}$ = $-n_1$, $-n_2$, $-n_3$ [see Eq. (3)].

For the F-axis, from Table 2, (2), (66), (14), (22), (68), (37) and Table B4, omitting Δ part energy of quarks, we have:

Table B6. The Energy Bands and the Quarks of the F-Axis (S = -1)

E_{Start}	E	E-Band	ΔS	J	S	C	ΔE	q(m $_{(\mathrm{Mev})}$)
$E_P=\frac{3}{4}$	583	(000)	1	0	0	0	0	d(313)
		(011,101)	1	1	0	0	0	$q_N^1(583)$
$E_P=\frac{3}{4}$	583	(110)	0	0	-1	0	0	$d_S^0(493)$
$E_H= 1$	673	(002)	0	0	-1	0	0	$d_S^0(773)$
		$(\bar{1}01,0\bar{1}1)$	1	2	0	0		$q_N^1(673)$
$E_P=\frac{11}{2}$	1303	(112)	0	0	-1	0	0	$d_S^0(1303)$
		$(1\bar{1}0,\bar{1}10)$	1	3	0	0		$q_N^1(1303)$
$E_P=\frac{11}{2}$	1303	$(01\bar{1},10\bar{1})$	1	4	0	0	0	$q_N^1(1303)$
		(020)	0	0	-1	0	-100	$d_S^0(1203)$
		(200)	0	0	-1	0	-100	$d_S^0(1203)$
		(121,211)	1	5	0	0	0	$q_N^1(1303)$
$E_H= 3$	1393	$(\bar{1}\bar{1}0)$	0	0	-1	0	0	$d_S^0(1393)$
		$(\bar{1}12,1\bar{1}2)$	1	6	0	0		$q_N^1(1393)$
$E_H= 3$	1393	$(\bar{1}\bar{1}2)$	0	0	-1	0	0	$d_S^0(1393)$
$E_P=\frac{19}{4}$	2023	$(0\bar{1}\bar{1},\bar{1}0\bar{1})$	-1	$J_{S,6}= 1$	-2	0	-200	$q_\Xi^{-1}(1823)$
		$(\bar{1}21)$	0	0	-1	0	-100	$d_S^0(1923)$
		$(2\bar{1}1)$	0	0	-1	0	-100	$d_S^0(1923)$
		(202, 022)	1	$J_C=1$	-1	1	-150	$q_{\Xi_C}(1873)$
$E_P=\frac{19}{4}$	2023	(220)	0	0	-1	0	0	$d_S^0(2023)$
		$(21\bar{1},12\bar{1})$	1	7	0	0		$q_N^1(2023)$
$E_H= 5$	2113	$(0\bar{2}0,\bar{2}00)$	-1	$J_{S,6}= 2$	-2	0	-200	$q_\Xi^{-1}(1913)$
		$(\bar{2}11)$	0	0	-1	0	-100	$d_S^0(2013)$
		$(1\bar{2}1)$	0	0	-1	0	-100	$d_S^0(2013)$
		(013,103)	1	$J_C=2$	-1	1	50	$q_{\Xi_C}(2163)$
$E_H= 5$	2113	$(\bar{2}\bar{1}1, \bar{1}\bar{2}1)$	-1	$J_{S,6}= 3$	-2	0	-200	$q_\Xi^{-1}(1913)$
		$(0\bar{2}2)$	0	0	-1	0	-100	$d_S^0(2013)$
		$(\bar{2}02)$	0	0	-1	0	-100	$d_S^0(2013)$
		$(0\bar{1}3,\bar{1}03)$	1	$J_C=3$	-1	1	250	$q_{\Xi_C}(2363)$
$E_P=\frac{27}{4}$	2743	$(2\bar{1}\bar{1},\bar{1}2\bar{1})$	1	$J_C=4$	-1	1	450	$q_{\Xi_C}(3193)$

$E_{Start} = [(n_1-\xi)^2+(n_2-\eta)^2+(n_3-\zeta)^2]$, E is the starting and minimum energy of the energy band; $\overline{n_i}$ = -n; $\overline{n_1 n_2 n_3}$ = -n_1, -n_2, -n_3 [see Eq. (3)].

For the G-axis, from Table 2, (2), (66), (14), (23),(69) and (37), omitting Δ part energy of quarks, we have:

Table B7. The Energy Bands and the Quarks of the G-Axis (S = -2)

E_{Start}	E	$(n_1n_2n_3, ...)$	ΔS	J	S	C	ΔE	q(m $_{(Mev)}$)
$E_N=\frac{1}{2}$	493	(000)	0	0	0	0	0	d(313)
		(110)	1	$J_{S,2}$=1	-1	0	0	d_S^1(493)
E_M= 1	673	$(101, 10\overline{1})$	0	0	-2	0	0	q_Ξ^0(673)
E_M= 1	673	$1\overline{1}0)$	1	$J_{S,2}$=2	-1	0	0	d_S^1(493)
		(200)	1	$J_{S,2}$=3	-1	0	100	d_S^1(773)
$E_N=\frac{3}{2}$	853	$(011, 01\overline{1})$	0	0	-2	0	0	q_Ξ^0(853)
$E_N=\frac{5}{2}$	1213	(020)	1	$J_{S,2}$=4	-1	0	-200	d_S^1(1413)
		$(\overline{1}10)$	1	$J_{S,2}$=5	-1	0	-300	d_S^1(1513)
E_M= 3	1393	$(0\overline{1}1, 0\overline{1}\overline{1})$	0	0	-2	0	0	q_Ξ^0(1393)
		$(211, 21\overline{1})$	0	0	-2	0	0	q_Ξ^0(1393)
E_M= 3	1393	$(2\overline{1}1, 2\overline{1}\overline{1})$	0	0	-2	0	0	q_Ξ^0(1393)
$E_N=\frac{7}{2}$	1573	$(\overline{1}01,\overline{1}0\overline{1})$	0	0	-2	0	0	q_Ξ^0(1573)
		$(121, 12\overline{1})$	0	0	-2	0	0	q_Ξ^0(1573)
$E_N=\frac{9}{2}$	1933	$(112, 11\overline{2})$	0	0	-2	0	-200	q_Ξ^0(1733)
		$(002, 00\overline{2})$	0	0	-2	0	-200	q_Ξ^0(1733)
		(220)	1	J_C=1	-2	1	200	d_{Ω_C}(2133)
		$(\overline{1}\overline{1}0)$	-1	$J_{S,6}$=1	-3	0	-300	d_Ω(1633)
E_M= 5	2113	$(202, 20\overline{2})$	0	0	-2	0	-200	q_Ξ^0(1913)
		$(1\overline{1}2, 1\overline{1}\overline{2})$	0	0	-2	0	-200	q_Ξ^0(1913)
		(310)	1	J_C=2	-2	1	400	q_{Ω_C}(2513)
		$(0\overline{2}0)$	-1	$J_{S,6}$=2	-3	0	-300	q_Ω(1813)
E_M= 5	2113	$(301,30\overline{1})$	0	0	-2	0	0	q_Ξ^0(2113)
		$(1\overline{2}1,1\overline{2}\overline{1})$	0	0	-2	0	0	q_Ξ^0(2113)
E_M= 5	2113	$(3\overline{1}0)$	1	$J_{S,2}$=6	-1	0	400	d_S^1(2513)
		$(2\overline{2}0)$	1	$J_{S,2}$=7	-1	0	500	d_S^1(2613)
$E_N=\frac{11}{2}$	2293	$(\overline{1}21,\overline{1}2\overline{1})$	0	0	-2	0	0	q_Ξ^0(2293)

E_{Start} = $[(n_1\text{-}\xi)^2+(n_2\text{-}\eta)^2+(n_3\text{-}\zeta)^2]$, E is the starting and minimum energy of the energy band; $\overline{n_i}$ = -n; $\overline{n_1n_2n_3}$ = $-n_1$, $-n_2$, $-n_3$ [see Eq. (3)].

Appendix C. Omitting Angular Momenta, Dividing Groups and Obtaining a Representative Particle of the Group

We show how to omit angular momenta and parities of the baryons or mesons and how to divide experimental baryons and mesons into groups using some examples in Table C1– C7. There might be many members in each group for experimental and deduced results. For a group with many members, we find a representative particle. The representative particle has the same intrinsic quantum numbers (I, S, C, b and Q) with the same name and the average rest mass of the members in the group. In Table C1, we show the representative particles of the deduced and experimental results. Similarly, we can get all representative baryons and mesons shown in Table C2 - Table C7.

The unflavored mesons with the same intrinsic quantum numbers but different angular momenta or parities have different names. In order to compare their rest masses, omitting the differences of the angular momenta and parities, we use meson η to represent the mesons with S = C = b = 0, I = Q = 0 (η, ϖ, ϕ, h and f) (see Table C5) and we use meson π to represent the mesons with S = C = b = 0, I = 1, Q = 1, 0, -1 (π, ρ, a and b) (see Table C6).

For example, the number ($\overline{1498}$) inside N($\overline{1498}$) is the rest mass of the baryon N($\overline{1498}$). The top line of the number ($\overline{1498}$) means that the number down the line is the average rest mass of the group. Γ is the full width of the baryon (or meson) in unit Mev, $\overline{\Gamma}$ means the average of the member full widths (Γs) in the group.

Table C1. The Unflavored Baryons N and Δ (S= C=b = 0)

Deduced	Experiment	Deduced	Experiment
$2N(1209)$ $2N(1299)$ $\mathrm{N}(\overline{1254})$		$2\Delta(1209)$ $2\Delta(1299)$ $\Delta(\overline{\mathbf{1254}})$	$\Delta(1232), 120$
$\mathbf{N(1479)}$	$N(1440), 350$ $N(1520), 120$ $N(1535), 150$ $\mathbf{N(\overline{1498}),\overline{207}}$		
$N(1659)$ $N(1659)$ $\mathbf{N}(\overline{\mathbf{1650}})$	$N(1650), 150$ $N(1675), 150$ $N(1680), 130$ $N(1700), 100$ $N(1710), 100$ $N(1720), 150$ $\mathbf{N(\overline{1689}),\overline{130}})$	$\Delta(1659)$ $\Delta(1659)$ $\boldsymbol{\Delta}(\overline{\mathbf{1659}})$	$\Delta(1600), 350$ $\Delta(1620), 150$ $\Delta(1700), 300$ $\boldsymbol{\Delta}(\overline{\mathbf{1640}}),\overline{\mathbf{267}}$
$2N(1839)$ $5N(1929)$ $2N(2019)$ $\mathbf{N}(\overline{\mathbf{1929}})$	$N(1900)^*$, 500 $N(1990)^*$, 500 $N(2000)^*$, 400 $N(2080)^*$, 400 $N(2090)^*$, 400 $\mathbf{N}(\overline{\mathbf{1912}})^*$, $\overline{440}$	$2\Delta(1929)$ $3\Delta(1929)$ $2\Delta(2019)$ $\boldsymbol{\Delta}(\overline{\mathbf{1955}})$	$\Delta(1905), 350$ $\Delta(1910), 250$ $\Delta(1920), 200$ $\Delta(1930), 350$ $\Delta(1950), 300$ $\boldsymbol{\Delta}(\overline{\mathbf{1923}}),\overline{\mathbf{264}}$
$N(2199)$ $N(2199)$ $\mathbf{N}(\overline{\mathbf{2199}})$	$N(2190), 450$ $N(2220), 400$ $N(2250), 400$ $\mathbf{N}(\overline{\mathbf{2220}}),\overline{\mathbf{417}}$	$\Delta(2379)$	$\Delta(2420), 400$
3N(2649)	**N(2600), 650**	$4\Delta(2649)$ $4\Delta(2739)$ $\boldsymbol{\Delta}(\overline{\mathbf{2694}})$	$\boldsymbol{\Delta}(\mathbf{2750})^*$,400
4N(2739)	**N(2700)*,600**		
1N(2919)	Prediction	$3\boldsymbol{\Delta}(\mathbf{3099})$	$\boldsymbol{\Delta}(\mathbf{2950})^*$,500 $\boldsymbol{\Delta}(\mathbf{3000})^*$,1000 $\boldsymbol{\Delta}(\overline{\mathbf{2975}})^*$, $\overline{750}$

Table C2. The Strange Baryons Λ and Σ (S = -1, C = b = 0)

Deduced	Experiment, Γ	Deduced	Experiment, Γ
$\mathbf{\Lambda}(\mathbf{1119})$	$\mathbf{\Lambda}(\mathbf{1116})$	$\mathbf{\Sigma}(\mathbf{1209})$	$\mathbf{\Sigma}(\mathbf{1193})$
$\Lambda(1399)$	$\Lambda(1406), 50$	$\Sigma(\mathbf{1399})$	$\mathbf{\Sigma}(\mathbf{1385}), \mathbf{37}$
$\Lambda(1659)$ $\Lambda(1659$ $\Lambda(1659)$ $\mathbf{\Lambda}(\overline{\mathbf{1659}})$	$\Lambda(1520), 16$ $\Lambda(1600), 150$ $\Lambda(1670), 35$ $\Lambda(1690), 60$ $\mathbf{\Lambda}(\overline{\mathbf{1620}}),\overline{\mathbf{65}}$	$\Sigma(1659)$ $\Sigma(1659)$ $\Sigma(1659)$ $2\Sigma(1829)$ $\mathbf{\Sigma}(\overline{\mathbf{1727}})$	$\Sigma(1660), 100$ $\Sigma(1670), 60$ $\Sigma(1750), 90$ $\Sigma(1775), 120$ $\mathbf{\Sigma}(\overline{\mathbf{1714}}),\overline{\mathbf{93}}$
$\Lambda(1829)$ $\Lambda(1829)$ $\Lambda(1929)$ $\Lambda(1929)$ $\Lambda(1929)$ $\mathbf{\Lambda}(\overline{\mathbf{1889}})$	$\Lambda(1800), 300$ $\Lambda(1810), 150$ $\Lambda(1820), 80$ $\Lambda(1830), 95$ $\Lambda(1890), 100$ $\mathbf{\Lambda}(\overline{\mathbf{1830}}),\overline{\mathbf{145}}$	$\Sigma(1929)$ $\Sigma(1929)$ $\Sigma(1929)$ $\mathbf{\Sigma}(\overline{\mathbf{1929}})$	$\Sigma(1915), 120$ $\Sigma(1940), 220$ $\mathbf{\Sigma}(\overline{\mathbf{1928}}),\overline{\mathbf{170}}$
$2\Lambda(2019)$ $\Lambda(2039)$ $\Lambda(2129)$ $\Lambda(2139)$ $\Lambda(2229)$ $\mathbf{\Lambda}(\overline{\mathbf{2095}})$	$\Lambda(2100), 200$ $\Lambda(2110), 200$ $\mathbf{\Lambda}(\overline{\mathbf{2105}}),\overline{\mathbf{200}}$	$\Sigma(2019)$ $\Sigma(2019)$ $\Sigma(2129)$ $\Sigma(\overline{\mathbf{2056}})$	$\Sigma(2000)^*, 200$ $\Sigma(2030), 180$ $\Sigma(2070)^*, 300$ $\Sigma(2080)^*, 200$ $\mathbf{\Sigma}(\overline{\mathbf{2045}}),\overline{\mathbf{220}}$
$\Lambda(2379)$	$\Lambda(2350), 150$	$\Sigma(2229)$	$\Sigma(2250), 100$
$2\Lambda(2549)$ $\Lambda(2559)$ $4\Lambda(2639)$ $4\Lambda(2649)$ $\mathbf{\Lambda}(\overline{\mathbf{2619}})$	$\mathbf{\Lambda}(\mathbf{2585})^*, 225$	$\mathbf{\Sigma}(\mathbf{2379})$ $2\Sigma(2549)$ $\Sigma(2492)$	$\Sigma(2455)^*, 140$
$5\Lambda(3099)$	Prediction	$4\Sigma(2639)$ $4\Sigma(2649)$ $\mathbf{\Sigma}(\overline{\mathbf{2644}})$	$\Sigma(2620)^*, 200$
$\Lambda(3369)$	Prediction	$5\mathbf{\Sigma}(\mathbf{3099})$	$\Sigma(3000)^*$, ? $\Sigma(3170)^*$, ? $\mathbf{\Sigma}(\overline{\mathbf{3085}})^*$, ?

Table C3. The Heavy Unflavored Mesons with S=C=b=Q=I=0

$d_S(9613)\overline{d_S(m)}$	E_{bind}	Deduced	Exper., Γ (Mev)	$\frac{\Delta M}{M}$%
$q_b^1(4913)\overline{q_b^1(4913)}$	- 576	$\Upsilon(9389)$	$\Upsilon(9460)$, 53 kev	0.75
$d_S^{-1}(9613)\overline{d_S^{-1}(493)_D}$ $d_S^{-1}(9613)\overline{d_S^0(493)_F}$	-250 -150	$\eta(9856)$ $\eta(9956)$ $\eta(\overline{9906})$	$\chi(9860)$ $\chi(9893)$ $\chi(9913)$ $\chi(\overline{9889})$	0.17
$d_S^{-1}(9613)\overline{d_S^1(493)_G}$ $d_S^{-1}(9613)\overline{d_S^1(493)_\Sigma}$	-50 -50	$\eta(10056)$ $\eta(10056)$	$\Upsilon(10023)$, 43 kev	0.13
$d_S^{-1}(9613)\overline{d_S^{-1}(773)_\Delta}$ $d_S^{-1}(9613)\overline{d_S^0(773)_\Delta}$ $d_S^{-1}(9613)\overline{d_S^1(773)_G}$	- 212 -112 -12	$\eta(10174)$ $\eta(10274)$ $\eta(10374)$ $\eta(\overline{10274})$	$\chi(10232)$ $\chi(10255)$ $\chi(10269)$ $\chi(\overline{10252})$	0.21
$u_C(6073)\overline{u_C(6073)}$	-1637	$\psi(10509)$	$\Upsilon(10355)$, 26 kev	1.5
$2d_S^{-1}(9613)\overline{d_S^0(1203)_F}$ $d_S^{-1}(9613)\overline{d_S^0(1303)_F}$ $d_S^{-1}(1503)\overline{d_S^{-1}(9613)}$	-241 -271 -431	$2\eta(10575)$ $\eta(10645)$ $\eta(10685)$ $\overline{\eta(10620)}$	$\Upsilon(10580)$, 20	0.38
$2d_S^{-1}(9613)\overline{d_S^0(1393)_F}$ $d_S^{-1}(1603)\overline{d_S^{-1}(9613)}$ $d_S^1(1413)\overline{d_S^{-1}(9613)}$ $d_S^1(1513)\overline{d_S^{-1}(9613)}$	-296 -461 -204 -234	$2\eta(10708)$ $\eta(10755)$ $\eta(10822)$ $\eta(10892)$ $\eta(\overline{10777})$	$\Upsilon(10865)$, 110	0.73
$d_S^0(1923)\overline{d_S^{-1}(9613)}$	-457	$\eta(11080)$	$\Upsilon(11020)$, 79	0.54
$4d_S^0(2013)\overline{d_S^{-1}(9613)}$ $d_S^0(2023)\overline{d_S^{-1}(9613)}$ $d_S^1(1933)\overline{d_S^{-1}(9613)}$	-483 -487 -360	$4\eta(11143)$ $\eta(11149)$ $\eta(11186)$ $\eta(\overline{11151})$	Prediction	

Table C4. The Strange Mesons (S = ±1, C = b = 0)

$q_N(313)\ \overline{d_S(m)}$	E_{Bind}	Deduced	Exper.
$q_N(313)\overline{d_S^{-1}(773)}$ $q_N(313)\overline{d_S^{0}(773)}$ $q_N(313)\overline{d_S^{1}(773)}$	- 170 - 270 - 170	K(916) K(816) K(916) K$(\overline{883})$	$K^{\pm}(892)$, 50
$2q_N(313)\overline{d_S^{0}(1203)}$ $q_N(313)\overline{d_S^{0}(1303)}$	-270 -270 -170	2K(1246) K(1346) K$(\overline{1279})$	K(1273), 90
$q_N(313)\overline{d_S^{0}(1393)}$ $q_N(313)\overline{d_S^{0}(1393)}$	-270 -270	K(1436) K(1436) K$(\overline{1436})$	$K_1(1402)$,174 $K^*(1414)$,232 $K_0^*(1412)$,294 $K_2^*(1429)$,100 $\overline{K(1414)}$, $\overline{200}$
$q_N(313)\overline{d_S^{1}(1413)}$ $q_N(313)\overline{d_S^{1}(1513)}$ $2q_N^0(1303)\overline{d_S^{\pm1}(493)}$ $3q_N^1(1303)\overline{d_S^{0}(493)}$	-170 -170 -190 -190	K(1556) K(1656) 2K(1606) 3K(1606) K$(\overline{1606})$	$K_2(1580)^{\#}$, 110 $K(1616)^{\#}$, 16 $K_1(1650)^{\#}$, 150 $K(\overline{1615})^{\#}$, $\overline{92}$
$q_N^1(1393)\overline{d_S^{0}(493)}$ $3q_N^1(1303)\overline{d_S^{-1}(493)}$ $\mathbf{q_N(313)}\overline{d_S^{-1}(\mathbf{1603})}$ $q_N^0(1573)\overline{d_S^{0}(493)}$ $q_N^1(1393)\overline{d_S^{-1}(493)}$ $q_N^0(1573)\overline{d_S^{1}(493)}$	-190 -90 -170 -290 -90 -190	K(1696) K(1706) K(1746) K(1776) K(1796) K(1876) K$(\overline{1766})$	$K^*(1717)$, 322 $K_2(1773)$, 186 $K_3^*(1776)$, 159 $K_2(1816)$, 276 K$(\overline{1771})$, $\overline{236}$
$q_N(313)\overline{d_S^{0}(1923)}$ $q_N(313)\overline{d_S^{0}(1923)}$	-270	K(1966)	$K_0^*(1950)^{\#}$, 201 $K_2^*(1973)^{\#}$, 373 $K(\overline{1962})^{\#}$, $\overline{287}$
$4q_N(313)\overline{d_S^{0}(2013)}$ $q_N(313)\overline{d_S^{0}(2023)}$ $q_N(313)\overline{d_S^{1}(1933)}$	-270 -270 -170	**4K(2056)** K(2066) K(2076) K$(\overline{2066})$	$K_4^*(2045)$, 198

Table C5. The Light Unflavored Mesons (S = C = b = 0) I = 0

$q_N(313)\overline{q_N(m)}$	E_{bind}	Deduced	Experiment, Γ	$\frac{\Delta M}{M}$%
$q_S(493)\overline{q_S(493)}$	-437	$\eta(549)$	$\eta(548)$, 1.29	0.4
$q_N(313)\overline{q_N(583)}$ $q_N(583)\overline{q_N(583)}$ $q_\Sigma(583)\overline{q_\Sigma(583)}$ $q_\Delta(673)\overline{d_\Delta(673)}$ $q_\Xi(673)\overline{d_\Xi(673)}$	-320 -528 -676 -938 -538	$\eta(576)$ $\eta(640)$ $\eta(490)$ $\eta(408)$ η (808) η ($\overline{584}$)	η(400-1200) Γ=600-1000	/
$q_N(313)\overline{q_N^1(583)}$ $q_N(313)\overline{q_N^1(673)}$ $q_N(673)\overline{q_N(673)}$ $q_N(313)\overline{q_N^0(853)}$	-220 -220 -538 -320	$\eta(676)$ $\eta(766)$ η (808) $\eta(846)$ $\eta(\overline{774})$	$\varpi(783)$, 8.49	1.15
$d_S^1(493)\overline{d_S^1(773)}$	-290	$\eta(976)$	$\eta'(958)$, 0.202 $f_0(980)$, 70. $\eta(\overline{969})$, $\overline{35}$	0.93
$d_S^1(773)\overline{d_S^1(773)}$ $d_S^0(773)\overline{d_S^0(773)}$ $d_S^{-1}(773)\overline{d_S^{-1}(773)}$	-505	$\eta(1041)$	$\phi(1020)$,4.26.	2.1
$d_S^1(493)\overline{d_S^{-1}(773)}$ $d_S^1(493)\overline{d_S^0(773)}$ $2q_N(313)\overline{q_N^0(1213)}$	- 90 -190 -320	$\eta(1176)$ $\eta(1077)$ $2\eta(1206)$ $\eta(\overline{1166})$	$h_1(1170)$, 360	0.26
$\mathbf{q_N(313)\overline{q_N^0(1303)}}$ $q_\Sigma(1033)\overline{q_\Sigma(1033)}$	**-320** -758	η**(1296)** $\eta(1308)$	$f_2(1275)$, 185 $f_1(1282)$, 24 $\eta(1294)$,55 $\eta(\overline{1284})$, $\overline{88}$	0.94
$q_N(313)\overline{q_N^1(1303)}$ $q_N(313)\overline{q_N^1(1303)}$	-220 -220	η(1396) η(1396)	$f_0(1350)$, 350 $\eta(1410)$, 51 $\varpi(1425)$, 215 $f_1(1426)$, 55 $\eta(\overline{1403})$, $\overline{168}$	0.50

Table C5 (Continuation). The Light Unflavored Mesons I = 0

$q_N(313)\overline{q_N(m)}$	E_{bind}	Deduced	Experiment
$q_N(313)\overline{q_N^1(1393)}$ $q_N(313)\overline{q_N^0(1573)}$	-220 -320	$\eta(1486)$ $\eta(1566)$ $\eta(\overline{1526})$	$\eta(1476)$, 87 $f_0(1507)$, 109 $f'_2(1525)$, 73 $\eta(\overline{1503})$, $\overline{90}$
$d_S^1(493)\overline{d_S^0(1303)}$ $2d_S^1(493)\overline{d_S^0(1393)}$ $d_S^1(493)\overline{d_S^1(1413)}$ $d_S^1(493)\overline{d_S^1(1513)}$	-190 -190 -290 -290	$\eta(1606)$ $2\eta(1696)$ $\eta(1616)$ $\eta(1716)$ $\eta(\overline{1666})$	$\eta_2(1617)$, 181 $\varpi(1670)$, 315 $\varpi_3(1667)$, 168 $\phi(1680)$, 150 $f_0(1714)$, 140 $\eta(\overline{1670})$,$\overline{191}$
$4d_S^0(1203)\overline{d_S^0(1203)}$ $d_S^1(493)\overline{d_S^{-1}(1503)}$ $4d_N^0(1303)\overline{d_N^0(1303)}$	-601 -90 -680	$4\eta(1806)$ $\eta(1906)$ $4\eta(1926)$ $\eta(\overline{1870})$	$\phi_3(1854)$, 87 $f_2(1945)$, 163 $\eta(\overline{1899})$, $\overline{125}$
$d_S^1(493)\overline{d_S^{-1}(1603)}$ $\mathbf{q_N(313)}\overline{q_N^0(2023)}$	-90 -320	$\eta(2006)$ $\eta\mathbf{(2016)}$	$f_2(2011)$, 202 $f_4(2034)$, 222 $\eta(\overline{2023})$, $\overline{212}$
$q_N(1393)\overline{q_N^0(1393)}$ $\mathbf{4q_N(313)}\overline{q_N^1(2023)}$ $d_S^1(1413)\overline{d_S^1(1413)}$	-707 -220 -663	$\eta(2079)$ $\mathbf{4}\eta\mathbf{(2116)}$ $\eta(2163)$	$f_0(2103)^{\#}$,206 $f_2(2156)^{\#}$,167 $\eta(\overline{2130})^{\#}$, $\overline{187}$
$q_N(313)\overline{q_N(2293)}$ $\mathbf{4d_S^1(493)}\overline{d_S^0(2013)}$ $d_S^1(493)\overline{d_S^0(2023)}$	-320 **-190** -190	$\eta(2286)$ $\mathbf{4}\eta\mathbf{(2316)}$ $\eta(2326)$	$f_2(2297)$, 149 $f_2(2339)$, 319 $\eta(\overline{2318})$, $\overline{234}$
$q_N^0(1573)\overline{q_N^0(1573)}$ $d_S^{-1}(1603)\overline{d_S^{-1}(1603)}$	-768 –728	$\eta(2378)$ $\eta(2478)$ $\eta(\overline{2428})$	$f_6(2465)^{\#}$, 255 .

Table C6. The Light Unflavored Mesons (S=C=b=0) with I = 1

$q_i(m_i)\overline{q_j(m_j)}$	E_{bind}	Phenomen.	Experiment, Γ
$q_N(313)\overline{q_N(673)}$	-487	$\pi(139)$	$\pi(138)$
$d_S^1(493)\overline{d_\Sigma^0(583)}$ $d_S^0(493)\overline{d_\Sigma^0(583)}$ $d_S^{-1}(493)\overline{d_\Sigma^0(583)}$	-267 -367 -267 $\overline{-300}$	$\pi(808)$ $\pi(708)$ $\pi(808)$ $\pi(\overline{775})$	$\pi(776)$, 150
$q_N^0(313)\overline{q_\Delta^0(673)}$	-19	$\pi(\overline{967})$	$a_0(985)$, 75
$d_S^1(493)\overline{d_\Sigma^0(1033)}$ $d_S^0(493)\overline{d_\Sigma^0(1033)}$ $d_S^{-1}(493)\overline{d_\Sigma^0(1033)}$	-268 -368 -268	$\pi(1258)$ $\pi(1158)$ $\pi(1258)$ $\pi(\overline{1225})$	$b_1(1230)$, 142 $a_1(1230)$, 425 $\pi(\overline{1230})$,$\overline{284}$
$2q_N^0(313)\overline{q_\Delta^0(1033)}$	-19	$\pi(\overline{1327})$	$\pi(1300)$, 400 $a_2(1318)$, 107 $\pi_1(1376)$, 300 $\pi(\overline{1331})$, $\overline{269}$
$\mathbf{d_S^1(493)}\overline{d_\Sigma^0(1303)}$ $d_S^0(493)\overline{d_\Sigma^0(1303)}$ $d_S^{-1}(493)\overline{d_\Sigma^0(1303)}$	-268 -368 -268	$\pi(1528)$ $\pi(1428)$ $\pi(1528)$ $\pi(\overline{1495})$	$\rho(1465)$, Γ=400 $a_0(1474)$,Γ=265 $\pi(\overline{1470})$, $\overline{333}$
$q_N^0(313)\overline{q_\Delta^0(1393)}$	-19	$\pi(\overline{1687})$	$\pi_1(1596)$, 312 $\pi_2(1672)$, 259 $\rho_3(1689)$, 161 $\rho(1720)$, 250 $\pi(\overline{1669})$, $\overline{246}$
$q_N(1213)\ \overline{q_N(1213)}$ $d_S^0(493)\overline{d_\Sigma^0(1753)}$	-654 -368	$\eta(1772)$ $\pi(1878)$ $\pi(\overline{1825})$	**π(1812), 207** $\rho(1900)^*$, 29
$d_S^1(493)\overline{d_\Sigma^0(1753)}$ $d_S^{-1}(493)\overline{d_\Sigma^0(1753)}$ $q_N^0(313)\overline{q_\Delta^0(1753)}$	-268 -268 -19	$\pi(1978)$ $\pi(1978)$ $\pi(2047)$ $\pi(\overline{2001})$	$\rho_3(1990)^*$,188 **a_4(2010), 353**
$3d_S^1(493)\overline{d_\Sigma^0(2023)}$	-268	$\pi(\overline{2248})$	$\rho_3(2250)^*$, 200
$4q_N^0(313)\overline{q_\Delta^0(2113)}$	-19	$\pi(2407)$	$a_6(2450)$, 400

Table C7. The Charmed Strange Mesons (S = ± 1, C = b = 0)

$u^1_C(m)\overline{d_S(493)}$	E_{bind}	Deduced	Experiment	$\frac{\Delta M}{M}\%$
$u^1_C(1753)\overline{d^1_S(493)}$	-303	$D_S(1943)$	$D^{*\pm}_S(1968)$	1.3
$u^1_C(1753)\overline{d^0_S(493)}$ $u^1_C(1753)\overline{d^{-1}_S(493)}$	-203 -103	$D_S(2043)$ $D_S(2143)$ $D_S(\overline{2093})$	$D_S(2112)$	0.90
$u^1_C(1753)\overline{d^1_S(773)}$	-215	$D_S(2311)$	$D_{S_j}(2317)$	0.04
$u^1_C(1753)\overline{d^0_S(773)}$ $u^1_C(2133)\overline{d^1_S(493)}$ $u^1_C(1753)\overline{d^{-1}_S(773)}$	-115 -184 -15	$D_S(2411)$ $D_S(2442)$ $D_S(2511)$ $D_S(\overline{2455})$	$D_{S_j}(2460)$	0.20
$u^1_C(2133)\overline{d^0_S(493)}$ $u^1_C(2333)\overline{d^1_S(493)}$	-84 -174	$D_S(2542)$ $D_S(2652)$ $D_S(\overline{2597})$	$D_{S_1}(2535)$ $D_{S_j}(2573)$ $D_{S_j}(\overline{2554})$	1.70
$u^1_C(2533)\overline{d^1_S(493)}$	- 164	$D_S(2862)$	Prediction	

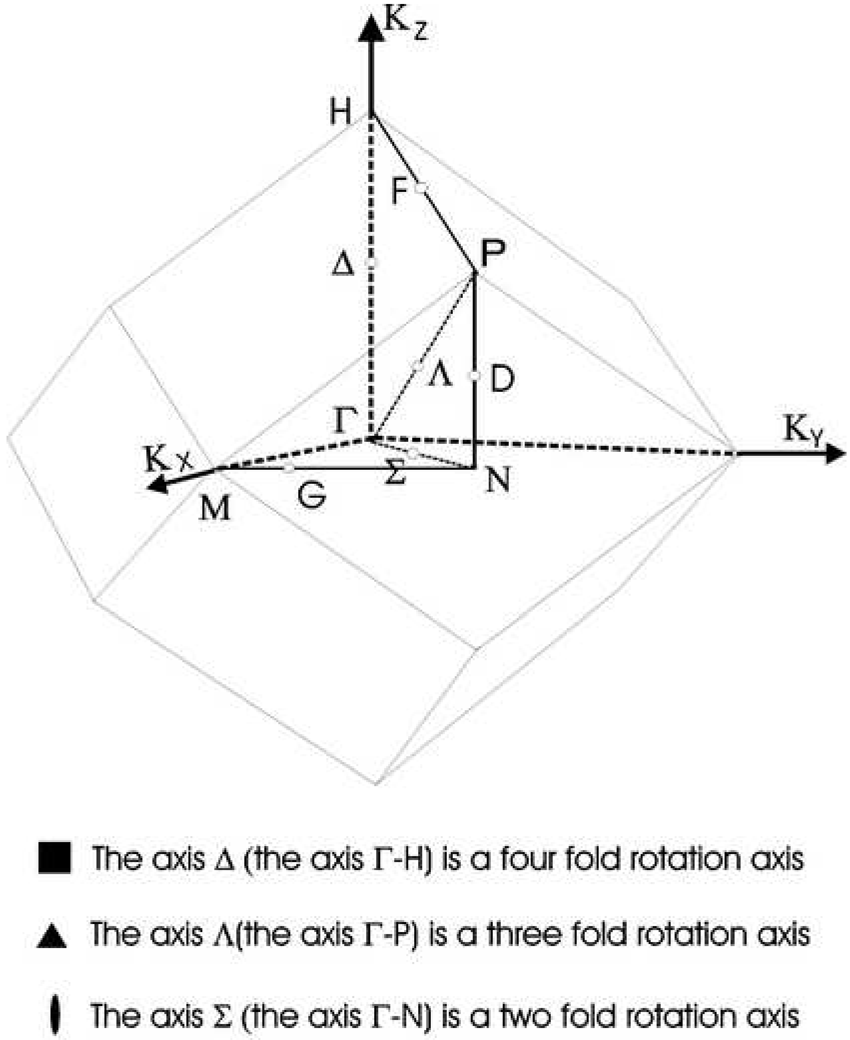

Figure 1

Figure 1: The regular rhombic dodecahedron. The symmetry points and axes are indicated. The Δ-axis is a fourfold rotation axis, the strange number S = 0 and the fourfold baryon family Δ (Δ^{++}, Δ^{+}, Δ^{0}, Δ^{-}) will appear on the axis. The axes Λ and F are threefold rotation axes, the strange number S = -1 and the threefold baryon family Σ (Σ^{+}, Σ^{0}, Σ^{-}) will appear on the axes. The axes Σ and G are twofold rotation axes, the strange number S = -2 and the twofold baryon family Ξ (Ξ^{0}, Ξ^{-}) will appear on the axes. The axis D is parallel to the axis Δ, S = 0. And it is a twofold rotation axis, the twofold baryon family N (N^{+}, N^{0}) will be on the axis.